THE BROADWAY SERIES OF ENGINEERING HANDBOOKS

TOOTHED GEARING

GEO. T. WHITE, B.Sc. (LOND)

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THE BROADWAY SERIES OF ENGINEERING HANDBOOKS
VOLUME IV

TOOTHED GEARING

BY

GEO. T. WHITE, B.Sc. (Lond.)

WITH 136 ILLUSTRATIONS

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PREFACE.

This handbook is an endeavour to make clear the action of the principal forms of teeth used in toothed gearing, together with other things of interest and importance in connexion with toothed wheels.

The two forms of tooth profile in most common use, namely the cycloidal and the involute, have been considered in detail, and neither strongly advocated in preference to the other. On cut gears the involute curve is more easily handled than the cycloids, and has almost entirely displaced them for this purpose, so much so that the involute tooth may very shortly become standardized for machine-cut teeth; the matter resting largely upon the agreement of the chief manufacturers of gear-cutting machines and the Institutions of Mechanical Engineers in this country and America.

A section on gear-cutting was intended to be included in this book, but it was found that to give it a reasonable treatment would double the size of the volume, so that it has been reserved for separate consideration.

Considering the growing importance of wheel

teeth in connexion with the reduction of high speeds of rotation of steam turbines and electric motors to more convenient speeds for general use, and the tendency towards all gear heads for machine tools, also the great use that is made of toothed gearing in motor-car driving, it is thought that a little book setting forth the essentials of toothed gearing might prove useful and acceptable to many who have not had the time and opportunity to study them through the usual channel of technical schools or books on mechanics. I also take this opportunity of thanking Mr. E. S. Andrews, B.Sc., for his suggestions and reading of the proofs.

GEO. T. WHITE.

LONDON, *July*, 1912.

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CHAPTER I

KINEMATICS

1. Kinematic Transformations.—To evolve a four-bar motion out of a simple bolt and nut by a succession of altered dimensions may seem strange or even impossible to one who has not studied the science of kinematics; yet the evolution is not difficult and the process may be followed by anyone not possessing a knowledge of that science.

The series of sketches, Figs. 1 to 15, taken in order, illustrate the changes.

Fig. 1 shows the simple bolt and nut.

Fig. 2 the simple bolt but the nut cut in halves.

Fig. 3 the simple bolt with the nut still further cut down.

Fig. 4 the simple bolt with the piece of nut bent. The transformation at this stage makes use of the fact that a straight line may be considered as the arc of a circle of infinite radius; the infinite radius of the straight piece of nut of Fig. 3 is made finite in Fig. 4.

Fig. 5 shows the curved piece of nut increased in length sufficiently to form a complete ring, the bolt retaining its old dimensions and position. By extending the material of the ring to the centre and mounting it on a spindle the nut has become endless, and a worm and worm wheel exists.

Fig. 6 shows the form of the thread changed from vee to approximately square.

Fig. 7 the diameter of the worm increased, and Fig. 8 the number of threads on the worm multi-

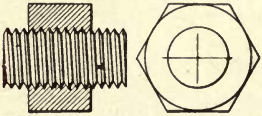


Fig 1

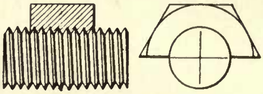


Fig 2

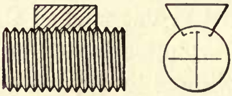


Fig 3

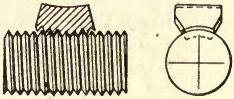


Fig 4

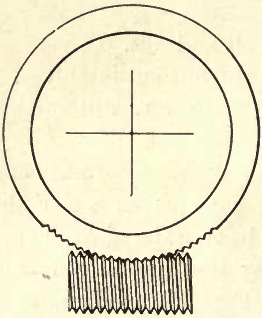


Fig 5

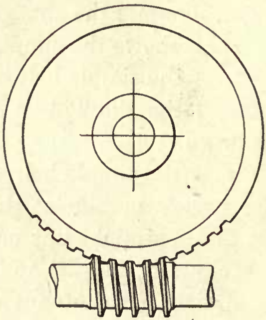


Fig 6

plied and the pitch increased, and the thread (now teeth) of the wheel altered to suit.

The form now is evidently that of a pair of screw wheels with axes at right angles.

Fig. 9 shows the wheels with their axes inclined

at 45°, the changing inclination of axis being accompanied by a change in the inclination of the teeth of one wheel across its rim.

Fig. 10 illustrates a further change in the inclina-

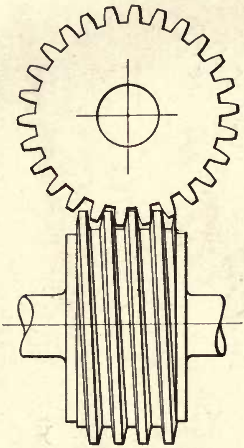


Fig.7.

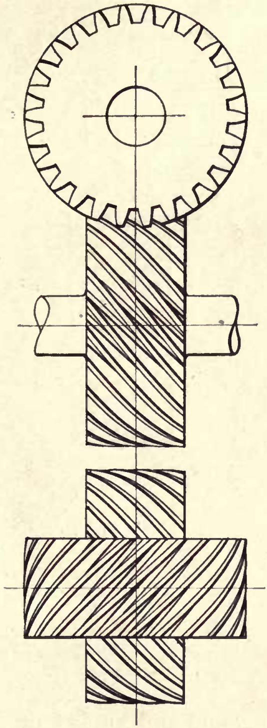


Fig.8.

tion of the axes. They have now become parallel, and, the worms (or teeth) of the other wheel being brought parallel to the axis, a pair of spur wheels have made their appearance.

Fig. 11 shows that portion of each wheel which

has contact with its neighbour. In this condition the amount of motion is limited.

Fig. 12 shows one tooth on each wheel formed as a portion of a round pin, and Fig. 13 the pins com-

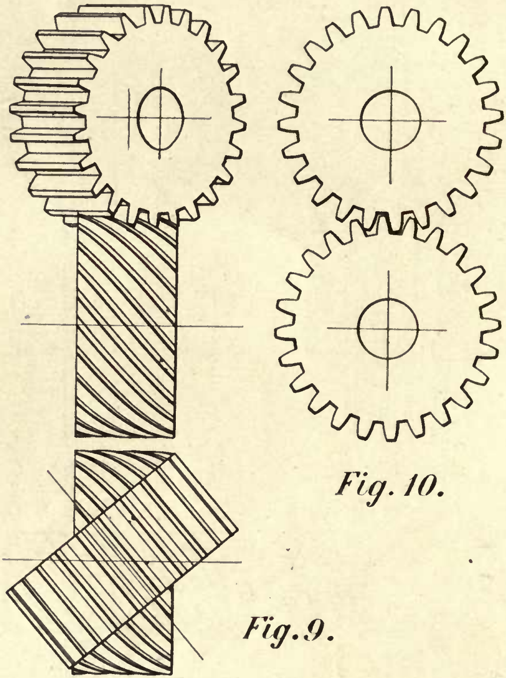


Fig. 10.

Fig. 9.

plete and surrounded by a link end, the link transmitting the thrust in the same direction as the previous arrangement, viz. Fig. 12.

Fig. 14 shows the thrust transmitting link much elongated but still acting exactly as in the two previous arrangements.

It is now in a very general form of a four-bar motion, and this form may be looked upon as the simplest of all mechanisms since it can be made with each of its four links exactly alike.

During these changes of form each important alteration has endowed the arrangement with new capabilities and properties while at the same time robbed it of others.

2. Changing Properties during Transformations.—From 1 to 3 no important change has occurred, but at 4 the curving of the nut makes an

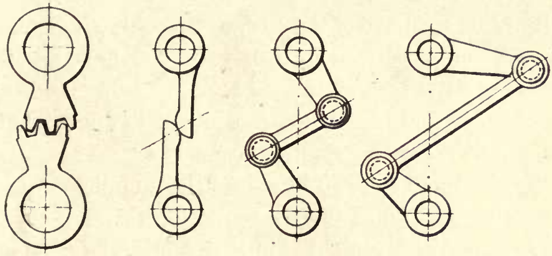


Fig. 11. *Fig. 12.* *Fig. 13.* *Fig. 14.*

infinite radius finite and the arrangement possesses, in a limited manner, the properties of a worm and worm wheel.

Figs. 4 to 7 are all capable of the same kind of movement: but at 8 the introduction of multiple threads provides an arrangement by which, with certain restrictions, any velocity ratio of the two shafts can be obtained independently of the wheel diameters and axis angles. 9 is in an intermediate state and at 10 the special properties of 8 have disappeared; by the axes becoming parallel the velocity ratio of the two wheels is fixed and is inversely proportional to their

diameters. If at 9 the teeth of the upper wheel had not been parallel to the axis, then the change to 10 would have produced a pair of helical spur wheels having the same properties as the straight spurs with respect to angular movement.

Fig. 11 is the same as 10 except that the working part being cut down the range of action is small. 12 has its surfaces of contact of cylindrical form and is the same as 11 for a still shorter range, theoretically for one instant only; for although the two arms may move in contact through a fair angular distance their angular velocity ratio will be changing all the time. The alteration in shape of the surfaces in contact has taken away the important property of constant angular velocity ratio possessed by 11.

Figs. 13 and 14 are modifications of 12 and are kinematically no further removed from 11 than 12 is; they are introduced here to illustrate the complete change from nut and bolt to four-bar motion.

If one member of this four-bar motion be fixed and the other three be made to rock about, the movements of the three free links are very definite and can be exactly determined.

3. The Four-bar Motion.—Referring to Figs. 15 and 16 which are diagrammatic views of the four-bar motion, AB is the fixed link, AC and BD are free to turn about A and B respectively, and CD may move about as constrained by the movements of the arms, its chief function being the transmission of motion from one arm to the other. CD is called the coupler or drag link. The introduction of the link CD between the points C and D may be looked upon as a convenient means of transmitting a force from

one point to the other, and apart from frictional disturbances the direction of the force transmitted will be along the line CD.

If the relative angular positions of AC and BD are not important the dimensions of AC, BD and CD can be altered. CD can be shortened until its length becomes zero and C touches D, and in this event, if the force between be a push, there is no need for the coupling rod CD; this is the fundamental case of the spur wheel and is illustrated at Fig. 11. The picture there shews contact at a point only since it represents

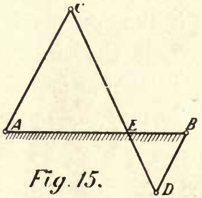


Fig. 15.

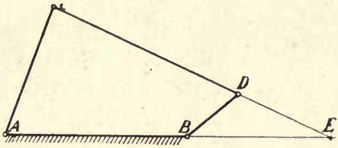


Fig. 16.

a plane section, but contact is in a line if the dimensions at right angles to that section be taken into account.

While the real link CD existed, contact was over the surfaces of the pins at C and D, but when the link disappeared its place was taken by a plane of thrust, and line contact came into being. The whole force transmitted being now concentrated along a line instead of being distributed over a surface, increased wear will ensue. This is always the case when one link of a mechanism is cut out: surface contact is displaced by line contact and the reduction of number of real links is paid for by increased wear and tear of the remaining ones.

4. Required Kinematic Property of Two Spur Wheels.—Now when two spur wheels are geared together the motion desired is one of constant angular velocity ratio; that is, assuming the driving wheel to already possess uniform angular velocity, the follower shall rotate likewise with a velocity equal to or some multiple of that of the driver; it shall not depart from that uniformity for the smallest fraction of a turn.

5. Angular Velocity in the Four-bar Motion.—To see how the conditions of paragraph 4 may be brought about, consider the angular velocity ratio of the two arms AC and BD in Figs. 15 and 16. The science of kinematics shows that if the coupler CD cuts the line of centres in E, then the angular velocity of AC is to that of BD in the proportion BE to AE; expressed in algebraic form—

$$\frac{\text{Ang. vel. of AC}}{\text{Ang. vel. of BD}} = \frac{\text{BE}}{\text{AE}};$$

or in words—the coupler divides the line of fixed centres in the inverse ratio of the angular velocities of the levers. Fig. 15 applies to external gearing and Fig. 16 to internal.

6. The Four-bar Motion a Pair of Spur Wheels.—From the equation of paragraph 5, for the arms AC and BD to possess a *constant* angular velocity ratio the coupler must always cut the line of fixed centres at the same spot, whether inside or outside. With the mechanism in the form of four solid bars this is impossible, for as soon as any movement occurs E simultaneously moves along AB; but if CD be displaced by a line of thrust only and C be brought into contact with D, by properly shap-

ing the surfaces of contact at C and D both arms may turn through a fair angular distance while still retaining contact, and during the whole of this period the line of thrust may pass through a fixed point on AB. Fig. 17 illustrates such an arrangement, the lettering of the diagram suiting the above description. If now the arms be provided with more than one pair of these surfaces, such that a new pair finds contact before an old pair loses it, the amount of

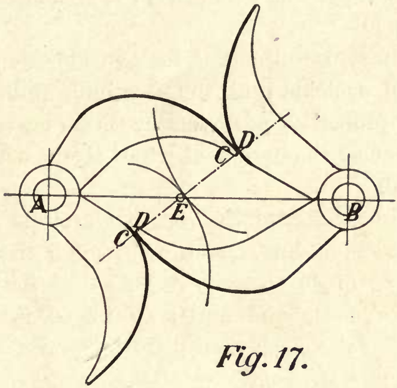


Fig. 17.

possible turning will be increased, and if the number of surfaces be sufficient and properly spaced complete rotation of each arm will be possible; and further, if a set of surfaces facing the opposite way be also fitted the rotation may take place in the reverse direction with equal accuracy. This is the case of a perfect spur wheel.

7. A Few Geometrical Facts.—The next step is to find the shape of the surfaces which, when in proper contact and transmitting a force, will ensure that such force shall pass through some fixed point.

Some facts to keep in mind during the search are: that apart from the influence of friction a surface reacts at right angles to itself at the place of contact, and that a normal to a curve at any point on it is a line at right angles to the tangent at that point: that when a plane section of a spur wheel is made at right angles to its axis all surfaces appear as lines and may be treated by all the methods of plane geometry; that all such plane sections of the same wheel are alike, and that the geometry for one holds for all.

The most useful curves for consideration are the family of cycloids and the involute, and the geometrical properties possessed by these curves, so far as they affect the design of wheel teeth, will be now described.

8. The Cycloidal Curves.—If a circle be rolled along a straight line, and a point upon the circumference be made to trace out its own path as the rolling proceeds, the curve so traced is called a *cycloid.* If the circle be rolled along on the convex side of some arc a point on the circumference of the rolling circle will trace out a curve known as an *epicycloid,* and if the rolling take place on the concave side of the arc the curve traced is known as a *hypocycloid.*

Consider the action of a circle rolling; imagine it as a wheel rolling along a straight level road. The action is one in which the wheel is constantly tumbling forward or turning as a whole about the point that is on the road; at any instant the point of contact with the road is stationary and the rest of the wheel turning about it as a centre, but as soon as

the least movement takes place a new point is on the road, the old one is off and turning about the new one; and so the wheel progresses, every point on the rim in turn coming to rest for an instant as it touches the road.

In Fig. 18 observe the motion of three points such

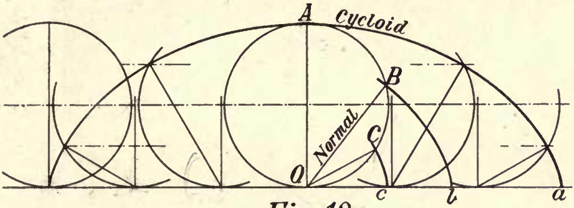


Fig. 18.

as A, B, and C: as the wheel rolls A traces the path A*a*, B the path B*b*, and C the path C*c*; each at the instant under consideration is turning about O and therefore describing circular arcs about O as centre; the lines OA, OB, and OC are the radii of circular arcs which for the instant are the paths of A, B, and C, and AO etc. are at right angles to those paths. One complete path for point *A* is shown in the figure, and the successive positions of the rolling circle indicated.

From this it will be seen that the normal at any point on a cycloid passes through that point and the point of contact of the rolling circle and its directing path, at the moment the chosen point is being generated.

Fig. 19 illustrates an epicycloid and a hypocycloid. The method of construction and the finding of the normals is the same as for the simple cycloid

with the exception that the directing path DD is curved.

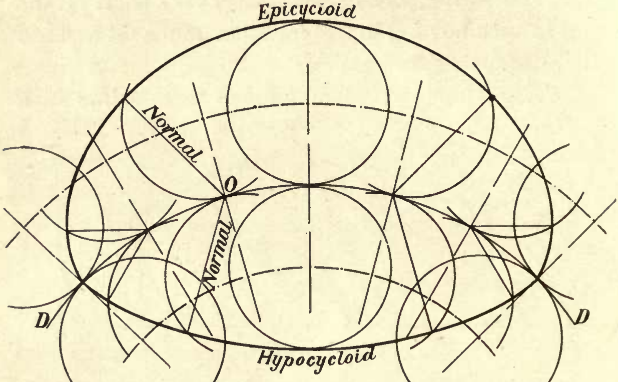


Fig. 19.

9. The Involute.—If a taut string without stretch be unwound from a curve and a point in the string be made to mark out its own path as the unwinding

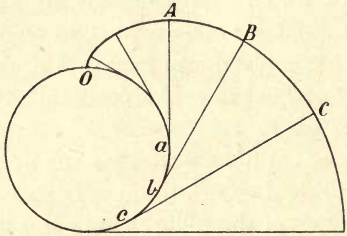


Fig. 20.

proceeds, the path so traced is the *involute* of the curve.

In Fig. 20, the track OABC is a part of the involute

of the circle O*abc*. The lines *a*A, *b*B, and *c*C are tangent to the circle and indicate different positions of a cord which started to unwind from O. The lengths *a*A, *b*B, etc., are equal to the circular arcs *a*O, *b*O, etc. At the instant the point A is being traced the end of the string is turning about *a* as a centre and is thus attempting a circular path of radius *a*A; therefore the line *a*A is the normal to the involute at A. Similarly *b*B is normal at B and *c*C normal at C.

CHAPTER II

THE SPUR WHEEL, CYCLOIDAL TEETH

10. Friction Wheel to Spur Wheel.—If two fairly smooth circular wheels mounted on axles are pressed together and one of them is made to rotate, the friction between the surfaces is sufficient to cause the second one to turn also, and provided the resistance to the movement of the second is not greater than the frictional grip can overcome then the one wheel will drive the other without slip, and the relative velocities of the two wheels will remain constant. If the resistance to movement is too great for the frictional grip then the second one refuses to move or movement occurs accompanied by slip. Roughing the surfaces will increase the frictional grip, but it is not a good practical way of overcoming the difficulty. If the surfaces of each be formed into a series of projections and hollows of regular pitch so that the projections of one wheel fall into the hollows of the other, then the force transmitted may be very great; even to the limit of nearly shearing off the projections. This introduces the spur wheel; the projections being the spurs or teeth.

In order that the relative angular velocities of the two wheels shall be constant, that is that the two wheels shall rotate together in as perfect a manner as two smooth wheels without slip, the kinematic condition pointed out in paragraph 6 must hold, viz. that the

normal to the tooth surfaces at the contact point must pass through some fixed spot on the line of centres.

Remember that any two curves that touch have a common tangent at their point of contact and consequently a common normal.

11. The Simple Spur Wheel.—In Fig. 21 let the two circles centres A and B represent two fairly smooth wheels capable of driving by friction, but that the force to be transmitted is too great for the frictional grip so that teeth must be provided. The surfaces originally

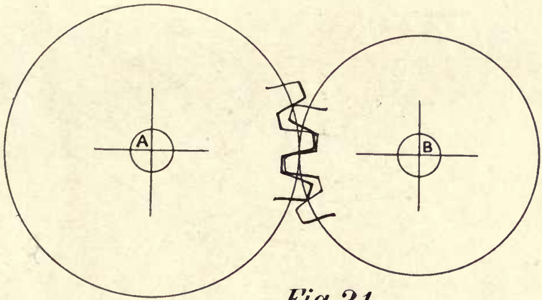


Fig. 21.

rubbing are the *pitch surfaces* of the spur wheels, and a section by a plane at right angles to the axes produces two circles in the figure. These circles are the *pitch circles* or *pitch lines*, their touching point is called the *pitch point* and is on the line of centres AB. When the teeth are provided this is the point through which the common normal must pass.

To preserve the identity of the original smooth wheels, the spur wheel may be looked upon as being built up by cutting pieces out of the original wheel at regular intervals, and putting the pieces so cut out upon the exterior, thus forming a greater

working projection or tooth, and trimming the working surfaces to satisfy the necessary conditions for the common normal. In practice the identity is always preserved in mind in as much as all important dimensions are reckoned from or upon the pitch line or circle, and the diameter of the pitch circle is the nominal diameter of the wheel.

12. Nomenclature of Spur Wheel Teeth.—The terms in common use applied to spur wheel teeth are illustrated in Fig. 22 and their definitions follow:—

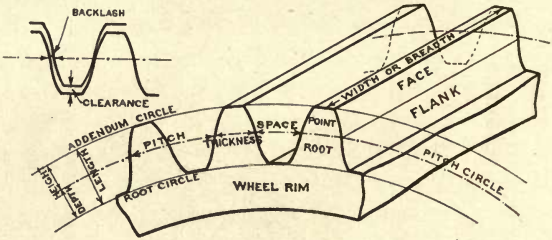


Fig. 22.

Pitch Surface, or *Pitch Cylinder* is the rim surface of the equivalent smooth wheel which working by friction will produce the same motion as the spur wheel.

Pitch circle is a plane section of the pitch surface taken at right angles to the axis of the wheel.

Pitch: circular or circumferential is the distance measured upon the pitch circle between two similar points on adjacent teeth; its dimension equals the length of the circumference of the pitch circle divided by the number of teeth on the wheel, or $\frac{\pi D''}{N}$, where D'' = wheel diameter and N = number of teeth.

Pitch : Diametral equals the number of teeth on the wheel divided by the diameter of the pitch circle, or $\frac{N}{D''}$; hence diametral pitch $= \frac{\pi}{\text{circular pitch}}$ and circular pitch $= \frac{\pi}{\text{diametral pitch}}$.

Pitch: *Module* equals the pitch diameter divided by the number of teeth on the wheel, or $\frac{D^{mm}}{N}$, or $\frac{D''}{N}$; originally used with metric measurements only.

The relation between the three systems of pitch measurement are expressed thus :—

$$\text{Module} = \frac{D^{mm}}{N} = \frac{\text{circular pitch}'' \times 25{\cdot}4}{\pi}$$

$$= \frac{25{\cdot}4}{\text{diametral pitch}} = \text{circular pitch}'' \times 8{\cdot}085.$$

Thickness equals the distance measured upon the pitch circle across the tooth itself.

Chordal thickness equals the length of the chord of the arc measured upon the pitch circle across the tooth itself.

Space equals the distance measured upon the pitch circle across the gap between two adjacent teeth.

Chordal space is measured in a similar manner to chordal thickness.

Point is the solid part of the tooth above the pitch surface.

Root is the solid part of the tooth below the pitch surface.

Width or *Breadth* is the distance from end to end of a tooth, or the width of the pitch surface.

Addendum circle or *Point circle* is the circle through the tops of the teeth.

Dedendum circle or *Root circle* is the circle through the bottom of the gaps between the teeth.

Height is the distance measured radially from the pitch circle to the addendum circle.

Depth is the distance measured radially from the pitch circle to the root circle.

Length is the radial distance from the root circle to the addendum circle; treating the tooth as a cantilever it is the amount of overhang.

Backlash is the amount of freedom of a tooth in its companion space, and equals the difference between space and thickness.

Clearance is the amount the top of a tooth falls short of the bottom of the gap into which it gears, it equals the difference between height and depth.

Face is the surface from the pitch cylinder to the outer edge of the tooth.

Flank is the surface from the pitch cylinder to the bottom of the gap.

The pitches as measured by the three systems do not convert from one to the other in round numbers, so that tables giving the equivalent pitches are found very convenient, and three such are here appended.

TABLES OF EQUIVALENT PITCHES

Circular Pitch ″.	Diametral Pitch.	Module. $\frac{D^{mm}}{N}$	Circular Pitch ″.	Diametral Pitch.	Module. $\frac{D^{mm}}{N}$
4	·786	32·34	$\frac{7}{8}$	3·590	7·075
$3\frac{3}{4}$	·838	30·31	$\frac{13}{16}$	3·867	6·570
$3\frac{1}{2}$	·898	28·30	$\frac{3}{4}$	4·189	6·065
$3\frac{1}{4}$	·967	26·28	$\frac{11}{16}$	4·570	5·555
3	1·048	24·26	$\frac{5}{8}$	5·027	5·050
$2\frac{3}{4}$	1·132	22·23	$\frac{9}{16}$	5·585	4·547
$2\frac{1}{2}$	1·256	20·21	$\frac{1}{2}$	6·283	4·043
$2\frac{1}{4}$	1·396	18·19	$\frac{15}{32}$	6·702	3·790
2	1·571	16·17	$\frac{7}{16}$	7·181	3·537
$1\frac{7}{8}$	1·676	15·16	$\frac{13}{32}$	7·734	3·285
$1\frac{3}{4}$	1·795	14·15	$\frac{3}{8}$	8·378	3·032
$1\frac{5}{8}$	1·933	13·14	$\frac{11}{32}$	9·140	2·778
$1\frac{1}{2}$	2·094	12·13	$\frac{5}{16}$	10·053	2·525
$1\frac{7}{16}$	2·185	11·62	$\frac{9}{32}$	11·170	2·274
$1\frac{3}{8}$	2·285	11·11	$\frac{1}{4}$	12·566	2·022
$1\frac{5}{16}$	2·394	10·61	$\frac{7}{32}$	14·362	1·769
$1\frac{1}{4}$	2·513	10·10	$\frac{3}{16}$	16·755	1·517
$1\frac{3}{16}$	2·646	9·595	$\frac{5}{32}$	20·106	1·263
$1\frac{1}{8}$	2·793	9·095	$\frac{1}{8}$	25·133	1·011
$1\frac{1}{16}$	2·957	8·590	$\frac{1}{10}$	31·416	·809
1	3·142	8·085	$\frac{3}{32}$	33·510	·759
$\frac{15}{16}$	3·351	7·580	$\frac{1}{16}$	50·266	·556

Diametral Pitch.	Module. $\frac{D^{mm}}{N}$	Circular Pitch ″.	Diametral Pitch.	Module. $\frac{D^{mm}}{N}$	Circular Pitch ″.
$\frac{3}{4}$	33·9	4·189	15	1·694	·209
1	25·4	3·142	16	1·587	·196
$1\frac{1}{4}$	20·32	2·153	17	1·495	·185
$1\frac{1}{2}$	16·93	2·094	18	1·411	·175
$1\frac{3}{4}$	14·51	1·795	19	1·338	·165
2	12·70	1·571	20	1·270	·157
$2\frac{1}{4}$	11·29	1·396	22	1·155	·143
$2\frac{1}{2}$	10·16	1·257	24	1·059	·131
$2\frac{3}{4}$	9·25	1·142	26	·977	·121
3	8·47	1·047	28	·907	·112
$3\frac{1}{2}$	7·26	·898	30	·847	·1047

Diametral Pitch.	Module. $\frac{D^{mm}}{N}$	Circular Pitch. ″	Diametral Pitch.	Module. $\frac{D^{mm}}{N}$	Circular Pitch ″.
4	6·35	·785	32	·794	·0982
4½	5·64	·698	34	·748	·0924
5	5·08	·628	36	·706	·0873
5½	4·62	·571	38	·669	·0827
6	4·23	·524	40	·635	·0785
7	3·63	·449	42	·606	·0748
8	3·175	·393	44	·578	·0714
9	2·822	·349	46	·553	·0683
10	2·540	·314	48	·529	·0654
11	2·309	·286	50	·508	·0628
12	2·117	·262	55	·462	·0572
13	1·954	·242	60	·424	·0524
14	1·814	·224			

Module. $\frac{D^{mm}}{N}$	Circular Pitch ″.	Diametral Pitch.	Module. $\frac{D^{mm}}{N}$	Circular Pitch ″.	Diametral Pitch.
0·5	·0618	50·8	10	1·236	2·540
0·75	·0927	33·87	11	1·360	2·309
1·0	·1236	25·40	12	1·484	2·117
1·25	·1545	20·32	13	1·606	1·954
1·5	·1855	16·93	14	1·731	1·814
1·75	·216	14·51	15	1·855	1·693
2·0	·247	12·70	16	1·98	1·587
2·25	·278	11·29	17	2·10	1·495
2·5	·309	10·16	18	2·23	1·411
2·75	·340	9·24	19	2·35	1·338
3·0	·371	8·47	20	2·47	1·270
3·5	·433	7·26	22	2·72	1·155
4·0	·495	6·35	24	2·97	1·059
4·5	·556	5·64	26	3·21	·977
5·0	·618	5·08	28	3·46	·907
5·5	·680	4·62	30	3·71	·847
6	·742	4·23	32	3·96	·794
7	·866	3·63	36	4·45	·706
8	·990	3·175	40	4·95	·635
9	1·112	2·822			

13. Tooth Proportions.—There is nothing to rigidly fix the various proportions of a tooth, though

certain given conditions may influence them; having determined a suitable section from considerations of strength, character of work to be performed and the mode of manufacture, the proportions used are those which experience has shown to be very serviceable.

A table of proportions in common use follows, the figures being in terms of the circular pitch.

TABLE OF TOOTH PROPORTIONS

	Pattern Moulded.	Machine Moulded.	Machine Cut.	Mortice Wheels.	Adcock's Proportions.	Browne & Sharpe Machine Cut.
Thickness	·47-·48	·485	·5	Cog. ·6; Iron tooth ·4	·48	·5
Space .	·53-·52	·515	·5	·4	·52	·5
Height .	·3-·35	·3-·35	·32	·25	·2	·318
Depth .	·35-·4	·35-·4	·36	·3	·3	·368

Clearance and backlash are obtained by difference, and length by addition. Breadth according to conditions: 2 to 4 times the pitch.

In systems using diametral pitches:—

$$\text{Height} = \frac{1}{\text{diametral pitch}} = \cdot 318 \text{ circular pitch.}$$

$$\text{Depth} = \frac{1}{\text{diametral pitch}} + \text{a clearance of } \frac{\cdot 157}{\text{diametral pitch}}.$$

Thickness and space as in the table for circular pitches.

In systems using module pitches:—

Height = module = ·318 Circular pitch″ = 8·085 Circular pitch$^{mm.}$

Depth = module + ·157 module.

Thickness and space as in the table for circular pitches.

14. The Tooth Profile, Cycloidal.—The working surfaces (face and flank) may be given the cycloidal or involute form, a small portion only of the curve being made use of; and in investigating their action it will be more convenient to consider a plane section than the whole surface of the tooth.

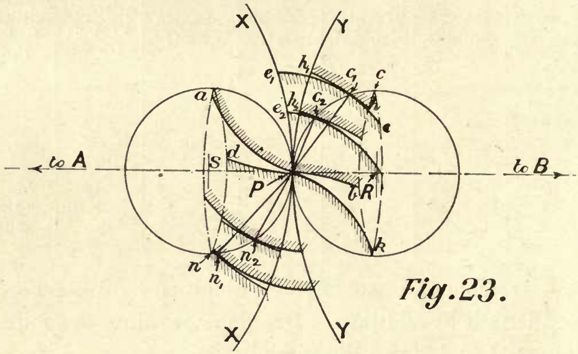


Fig.23.

In Fig. 23, the arcs XX and YY are parts of the pitch lines of two spur wheels centres at A and B; P is their point of contact, or pitch point; circle centre R touches both at P and is ready to roll on either. Selecting a point C_1 on the R circle and rolling the circle on YY the hypocycloid hh_1 will be traced out; rolling on XX the epicycloid ee_1 will be the result, and the lengths of the arcs Pc_1, Ph_1, and Pe_1 will be equal, since perfect rolling must be assumed. In the position drawn c_1P is the normal at c_1 for hh_1 and for ee_1 since the R circle touches the directing path of both at P; the two curves having a common normal at c_1 they

will be tangent there. If any other point such as c_2 be selected the same properties would exist, the common normal would pass through P, arcs Pc_2, Ph_2, and Pe_2 would be equal and the curves traced would be exactly like those from C_1.

If now the two pitch lines and the R circle all roll together, there being no slip, their common point of contact will always be at P; c_2, h_2 and e_2 will arrive at P together, so also will c_1, h_1 and e_1; thus c_2 may be looked upon as a new position of c_1 as it approaches P, and whatever the position of the two curves hh_1 and ee_1, within limits, they will have contact and the contact point will be on the circumference of the R circle. After passing P however they separate and if contact be desired beyond this point another pair of surfaces must be provided; a second pair is shown in the figure on the left and below P, the circle with centre S being the rolling circle. The S circle rolls an epicycloid on YY and a hypocycloid on XX. By joining the two pairs one pair of continuous curves is obtained which, with the lengths of curve in the figure, pick up contact at c and lose it at n, or motion may be in the reverse direction and contact commence at n and end at c. The combination is shown at aPb and dPk.

The R and S circles are drawn of equal diameter but this need not be; so long as two rolling circles are used, one on each side of the pitch point, and each rolls its pair of curves all is theoretically correct.

These curves can now be used as the profiles of the working surfaces of teeth, dP forming the flank and Pk the face of the teeth upon XX and aP the face and Pb the flank of the teeth upon YY, the face

of one tooth working against the flank of its companion. From the above consideration follows the rule that *the circle that rolls the face of a tooth must also roll the flank of its mate.*

For a pair of wheels to rotate in either direction, the same wheel being the driver in each case, it is necessary to provide working surfaces on both sides of the teeth, for the side of a tooth in contact with its mate changes on reversal of rotation. Also that a wheel may be fitted to its shaft without consideration of direction of rotation the teeth must have working surfaces on both sides. Obviously the simplest and easiest arrangement is to use the same curves for each side but looking opposite ways.

15. Profile for Interchangeable Set.—When a number of different sized wheels are required to work any one with any other and the range is large, such as a set of change wheels for a lathe, the cycloidal form of tooth is at a disadvantage; for the same rolling circle must be used throughout in the design of the teeth, since any face must work with any flank, and a rolling circle suitable for a large diameter wheel is not equally so for a small one of the same pitch; also the wheel centres must be an exact distance apart for accurate tooth action.

A rolling circle of one quarter the pitch circle will produce a good form. Considering wheels of 120 and 30 teeth a rolling circle of one quarter the diameter of the 120 wheel will equal the whole diameter of the 30 wheel; rolling this within the 30 wheel the hypocycloid becomes a point only and the tooth has no flank. The involute overcomes both diffi-

culties, tooth form and centre distance, and is discussed in paragraph 35, *et seq*.

16. Full, Radial, and Undercut Flanks.—When the rolling circle is half the diameter of the pitch circle the hypocycloid becomes a straight line passing through the centre of the wheel, and the teeth having their flanks formed under these conditions have what are known as radial flanks. When the rolling circle is greater than half the pitch circle the hypocycloid

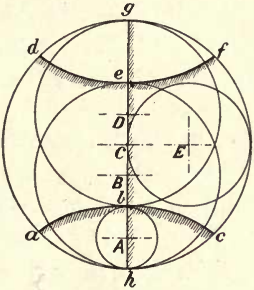


Fig. 24.

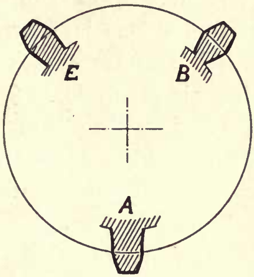


Fig. 25.

reverses its curvature and what are known as undercut flanks are produced.

Fig. 24 shows a few hypocycloids generated by different rolling circles. The large circle with centre C is the director circle. Rolling circle with centre A will produce *abc*, circle with centre B will give *def* and circle with centre E, *g*C*h*. Circle with centre D will also give *abc*, but then the curve is convex towards the point of contact of rolling (or generator) circle and the fixed (or director) circle. The shape of the flanks resulting from the use of these curves is seen in Fig. 25. A from the A circle, B from the B circle, and E from the E circle.

Since the comparative strength of teeth of the same material and proportions may be taken as the square of the thickness at the root, it is evident from Fig. 25 that the tooth at A is very superior to that at B, while the radial flanks at E hold an intermediate position but are still weak in comparison with those at A. Some strength is added to all the forms by putting a small radius (or fillet) at the junction of the tooth and the rim of the wheel.

When the pitch circle is of infinite diameter its circumference is a straight line and the toothed wheel becomes a rack; the profiles of the teeth are then simple cycloids.

17. Limiting Shapes for Standard Cycloidal Teeth.—Fig. 26 shows a 12-toothed pinion with radial flanks, and gearing with it on the under side a 120-toothed wheel and on the upper side a rack, the same rolling circle having been used throughout to generate the profiles. The difference between the shapes of the teeth of the wheel and the pinion is seen to be considerable, while that between the wheel and the rack is hardly perceptible. The figure serves to illustrate the two limits of the form of teeth of ordinary proportions, ·3 and ·4 pitch having been used for height and depth, and the rolling circle for faces and flanks half the diameter of the 12-toothed pinion; the assumption being that a pinion of 12 teeth is the smallest in general use, and is used as a standard by manufacturers of mills for gear cutting.

18. Arcs of Contact, Approach, and Recess.—For the condition to hold that in any position at least one pair of teeth have contact, the pitch of the

teeth must not be greater than the arc turned through during the period of contact.

In Fig. 27 A and B are the centres of the wheels, R and S those of the rolling circles. First and last contact of one pair of teeth is shown. First contact,

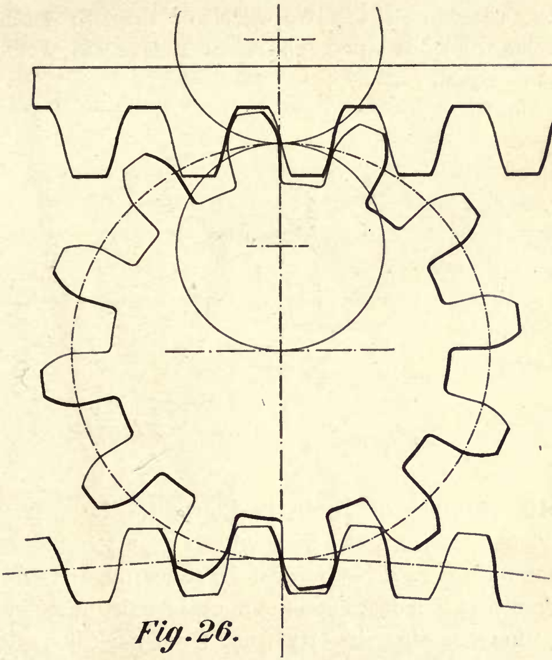


Fig. 26.

a, is found by the intersection of the addendum B circle with the R circle and last contact, *b*, by the intersection of the addendum A circle with the S circle. With contact at *a* the teeth profiles cut their respective pitch lines at *c* and *e*, with contact at *b* they cut at *d* and *f*. The arc *c*P*d* is the *arc of con-*

tact for A and ePf for B; the parts cP and eP are the *arcs of approach* and Pd and Pf the *arcs of recess;* and it should be noted that $cP = eP = aP$ and $dP = fP = bP$.

In the figure the arcs of approach and recess are equal because the two wheels are made equal in every respect. If the two wheels or the two rolling circles were not equal, approach and recess would not be equal.

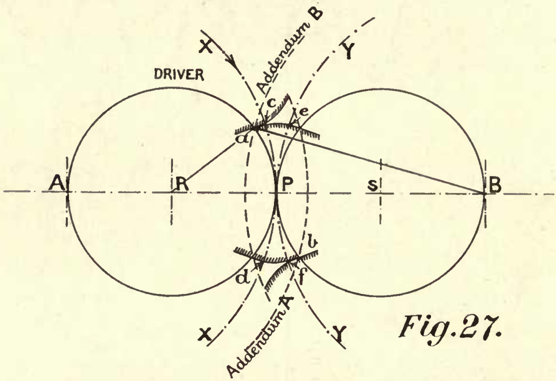


Fig. 27.

19. Number of Teeth in Contact.—It is quite obvious that when two spur wheels gear together at least one pair of teeth must be touching, and that before that pair loses contact another pair must find it, otherwise after the driver had freed itself it would rotate unchecked or the next pair of teeth would meet with a shock.

If the arc of contact be greater than the circular pitch, then there will always be contact between one or more pairs; if the arc be greater than twice the pitch, then two and for a short period three pairs will be in contact.

The ratio $\frac{\text{arc of contact}}{\text{circular pitch}}$ is called the *number of teeth in gear.*

20. Numerical Examples.—The investigation of the number in gear under different conditions can be made by the aid of a scale drawing, but by the application of the trigonometrical properties of triangles the results may be obtained with greater exactness without a drawing to scale.

Assuming the very common proportion height of tooth = ·3 pitch, and taking the limit of smallness for a pinion to be one of 12 teeth with radial flanks, and using the same rolling circle throughout, that is all curves rolled by a circle equal in diameter to half the pitch circle of a 12-teeth pinion, with 1 inch circumferential pitch the calculations are as follows:—

Pitch diam. of 12-teeth pinion $= \frac{12}{\pi} = 3{\cdot}82''$

Diam. of rolling circle $= \frac{3{\cdot}82}{2} = 1{\cdot}91''$

Radius of pitch circle $= \frac{3{\cdot}82}{2} = 1{\cdot}91''$

Radius of rolling circle $= \frac{1{\cdot}91}{2} = {\cdot}955''$

Radius of addendum circle $= 1{\cdot}91 + {\cdot}3 = 2{\cdot}21''$

Referring to Fig. 27 which was drawn for two equal wheels and radial flanks, from the triangle RaB the angle aRP can be calculated and from this angle the length of the arc aP; and since $d\text{P} = a\text{P}$ the arc of contact = twice aP.

$$\text{RB} = {\cdot}955 + 1{\cdot}91 = 2{\cdot}865.$$

The relations of the sides of the triangle RaB are given by the equation

$$aB^2 = aR^2 + RB^2 - 2aR \cdot RB \cos aRB,$$

substituting the known values

$$2{\cdot}21^2 = {\cdot}955^2 + 2{\cdot}865^2 - 2 \times {\cdot}955 \times 2{\cdot}865 \cos aRB,$$

$$\text{from which } \cos aRB = {\cdot}785'$$

$$\text{and angle } aRB = 38^\circ\ 18'$$

$$aP = {\cdot}955'' \times 2\pi \times \frac{38^\circ{\cdot}18^{1'}}{360^\circ} = {\cdot}638''$$

$$= \text{half the arc of contact,}$$

$$\text{whole arc of contact} = 2 \times {\cdot}638 = 1{\cdot}276''$$

$$= 1{\cdot}276 \text{ pitch.}$$

Retaining the same diameters of wheels and rolling circles and reducing the pitch to one half inch :—

$$\text{Height} = {\cdot}3 \text{ pitch} = {\cdot}3 \times {\cdot}5'' = {\cdot}15''$$

$$\text{Radius of addendum circle} = 1{\cdot}91 + {\cdot}15 = 2{\cdot}06''.$$

Substituting in the same equation as for one-inch pitch; remembering aB is now 2·06″

$$2{\cdot}06^2 = {\cdot}955^2 + 2{\cdot}865^2 - 2 \times {\cdot}955 \times 2{\cdot}865 \cos aRB$$

$$\text{from which } \cos aRB = {\cdot}891$$

$$\text{and angle } aRB = 27^\circ$$

$$aP = {\cdot}955'' \times 2\,\pi \times \frac{27^\circ}{360^\circ} = {\cdot}45''$$

$$= \text{half the arc of contact,}$$

$$\text{whole arc of contact} = {\cdot}9''$$

$$= 1{\cdot}8 \text{ pitch.}$$

21. Arc of Contact, Graphically.—Fig. 28 illustrates several cases. XX and YY are two equal pitch circles with centres A and B, R and S are centres for rolling circles equal to half the pitch circles, *ab* is the addendum circle for B and *dc* that for A. With A

driving as indicated on the diagram, 1 is first contact and 1_1 last contact, $1P = P1_1$. With the pitch reduced to half and retaining the same rolling circle and proportions for the teeth, point 2 is first contact and 2_1 last contact. It is clear from the picture that the arc P2 is greater than one half of the arc P1, which supports the calculations of the previous paragraph, and from which is obtained the fact that diminishing the

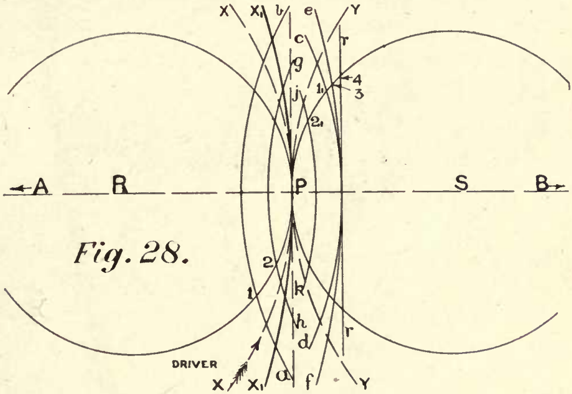


Fig. 28.

pitch of the teeth increases the ratio of arc of contact to pitch of teeth.

Next if the pitch diameter of the driver be increased to say twice its original size, with the other dimensions the same as in the numerical example, the arc of approach P1 remains as before ·638″ but the arc of recess, P3 in the figure, becomes ·704″, and when the driver enlarges to a rack the recess P4 = ·778. So that increase of wheel diameter also increases the ratio arc of contact to pitch of teeth.

Increase in diameter of rolling circle and a greater

height of tooth, which enlarges the addendum circle, both carry the point of first or last contact further from the pitch point, thus again increasing the arc of contact.

Altogether then, to obtain the greatest number of teeth in gear, a large wheel, small pitch of teeth, a high tooth, and large rolling circle should be used.

CHAPTER III

ANNULAR WHEELS, CYCLOIDAL TEETH

22. Internal Gears or Annular Wheels.—After the limit of infinity for the diameter, the rack bends, or the curvature of the wheel reverses; the centre of the wheel comes in from infinity on the opposite side and the wheel is now one with internal teeth. This introduces the subject of *Internal Gears* or *Annular Wheels.*

The tooth profiles of annular wheels may be obtained in the same way as for outside spurs, noting that when the pitch line reversed its curvature the outside became the inside, so that the epicycloid became a hypocycloid and the hypocycloid an epicycloid.

In Fig. 29 a 12-teeth pinion with centre A with radial flanks is driving a 24-teeth annular wheel with centre B. XX and YY are the pitch circles, the R circle traces the flank of the pinion and the face of the wheel tooth by rolling on XX and YY respectively, and the S circle traces the face of the pinion and flank of the wheel tooth. The intersection a of the addendum circle of B with the R circle is first contact, and the intersection b of the addendum circle of A with the S circle is last contact, the direction of motion being anti-clockwise. Assuming 1″ pitch the length aP = ·929″ and Pb = ·638″.

Thus far internal gearing possesses exactly the same properties as external gearing.

23. Velocity Ratio Limited.—Whereas in external gearing the ratio of the wheel diameters might be anything from one to infinity, with internal gearing having teeth of ordinary proportions there is not

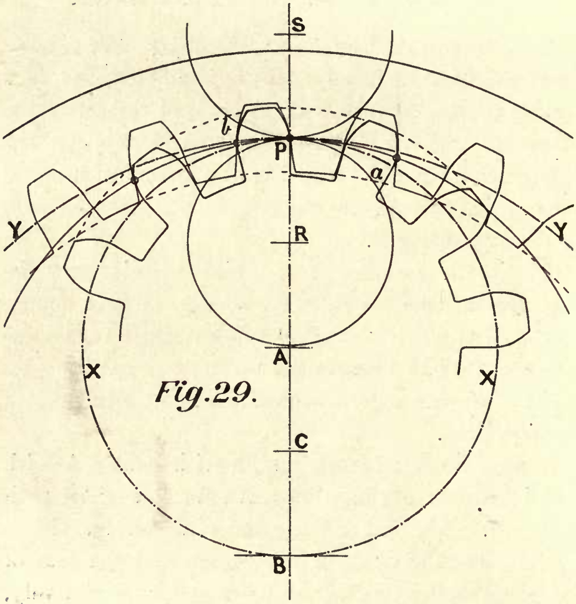


Fig. 29.

the same scope. Given a definitely sized pinion and a rack the ratio is infinity; if the rack be now curved with the teeth on the inside it becomes an annular wheel, and there is a finite ratio of wheel to pinion; reducing diameter of wheel reduces the ratio, but the reduction cannot go on until the ratio is unity, before that occurs the teeth will foul.

A knowledge of the minimum ratio and of the factors which affect it is important, and the investigation of them introduces the peculiar property of secondary contact possessed only by internal gears having cycloidal teeth.

24. Secondary Contact or Secondary Action.—

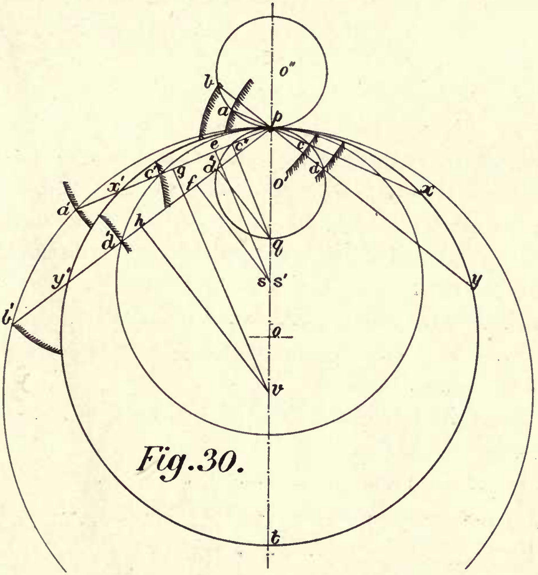


Fig. 30.

In Fig. 30 o'' is the centre of the external rolling circle and o' that of the internal rolling circle on pitch circle of diameter pt. b is a point on the epicycloidal tooth face shown and bp is its normal, which when produced meets the pitch circle again at y. If the tooth face move round to the left until y coincides with p, b will be at b', the whole normal by will occupy the position $b'p$, and since the normal passes

through the pitch point p, the point at b' is now equally ready to act as it was at b; and if it could meet a proper companion surface it would do so.

Similarly with the point a and its normal apx, when x arrives at p, a is at a', and ready to act. Also with the points c and d on the internal rolling circle and their normals pcx and pdy; when x arrives at p, c is at c', and when y arrives at p, d is at d', both c' and d' are ready to act.

25. Path of Secondary Contact.—These points a', b', c', d', are points of *secondary action*, and if a number of such points be plotted and a line drawn through them it will be found to be a circle in each case, both external and internal; the external one being equal to the sum of the pitch circle and external rolling circle, and the internal one equal to the difference of the pitch circle and internal rolling circle. A geometrical proof may be of interest and is here added.

26. Geometrical Proof of Path of Secondary Contact.—In Fig. 30:—

pt is the diameter of the pitch circle = D,
pq is the diameter of the internal rolling circle = D′,
$c''p = cp$ and $d''p = dp$ by symmetry,
$x'c' = cp = c''p$,
$y'd' = dp = d''p$,
point e bisects $c'p$ and f bisects $d'p$,
es is at right angles to $c'p$ and meets pt in s

$$\therefore ps = c's,$$

fs' is at right angles to $d'p$ and meets pt in s'

$$\therefore ps' = d's',$$

q is joined to c'' and d'', then $qc''p$ and $qd''p$ are both right angles being the angles in a semicircle;

$$\frac{x'p}{x'c'} = \frac{x'p}{c''p} = \frac{D}{D'} \text{ by proportion of similar figures}$$

$$\therefore x'p = x'c' \frac{D}{D'};$$

$$\frac{y'p}{y'd'} = \frac{y'p}{d''p} = \frac{D}{D'} \text{ by proportion of similar figures}$$

$$\therefore y'p = y'd' \frac{D}{D'};$$

$$ep = \tfrac{1}{2}c'p = \tfrac{1}{2}(x'p - x'c') = \tfrac{1}{2}(x'c'\frac{D}{D'} - x'c')$$

$$= x'c'\tfrac{1}{2}\left(\frac{D}{D'} - 1\right)$$

$$= x'c'\tfrac{1}{2}\left(\frac{D - D'}{D'}\right) \quad . . \text{(I)}$$

fp by similar reasoning $\quad = y'd'\tfrac{1}{2}\left(\frac{D - D'}{D'}\right) . . \text{(II)}$

Comparing the two similar triangles $c''pq$ and eps

$$\frac{ep}{ps} = \frac{c''p}{pq}$$

substituting for ep its value in I, and $x'c'$ for $c''p$, and D′ for pq

$$\frac{x'c' \times \tfrac{1}{2}\dfrac{D - D'}{D'}}{ps} = \frac{x'c'}{D'}$$

which reduces to $ps = \tfrac{1}{2}(D - D')$ . . (III)

Comparing the two similar triangles $d''pq$ and fps'

$$\frac{fp}{ps'} = \frac{d''p}{pq}$$

substituting for fp its value in II, and $y'd'$ for $d''p$, and D′ for pq, the equation becomes

$$\frac{y'd' \times \frac{1}{2}\frac{\mathrm{D} - \mathrm{D}'}{\mathrm{D}'}}{ps'} = \frac{y'd'}{\mathrm{D}'}$$

which reduces to $ps' = \frac{1}{2}(\mathrm{D} - \mathrm{D}')$ . . (IV)
From III and IV $ps = ps'$, therefore $c's = d's'$ and s and s' coincide.

If points other than c and d on the internal rolling circle be chosen and the corresponding points similar to c' and d' be found, they can be shown to be equally distant from s and that distance equal $\frac{1}{2}$ $(\mathrm{D} - \mathrm{D}')$; in other words they lie on a circle whose diameter is the difference of the diameters of pitch circle and rolling circle.

For the locus of external secondary action, bisect $a'p$ and $b'p$ at g and h and draw perpendiculars, they will meet at v on pt, and a similar reasoning to that above shows that

$$pv = \tfrac{1}{2}(\mathrm{D} + \mathrm{D}'')$$

where D'' = diameter of external rolling circle.

Thus the locus of external secondary contact is a circle whose diameter is the sum of the diameters of pitch circle and external rolling circle.

27. Secondary Contact in Annular Wheels.—The pitch circle chosen might have been that of an annular wheel if due regard had been taken to differentiate between faces and flanks, so that an annular wheel will have its circles of secondary contact.

Fig. 31 shews the pitch lines of a pinion and annular wheel gearing with it; it also shews the internal and external circles of secondary contact for both.

28. Conditions which produce Secondary Contact.—As drawn, no secondary contact occurs, for the

wheel is too large, but if the wheel gradually diminishes in diameter, the rolling circles remaining the same, the internal secondary circle of the wheel will eventually coincide with the external secondary of the pinion ; in which condition the *faces* of the wheel and

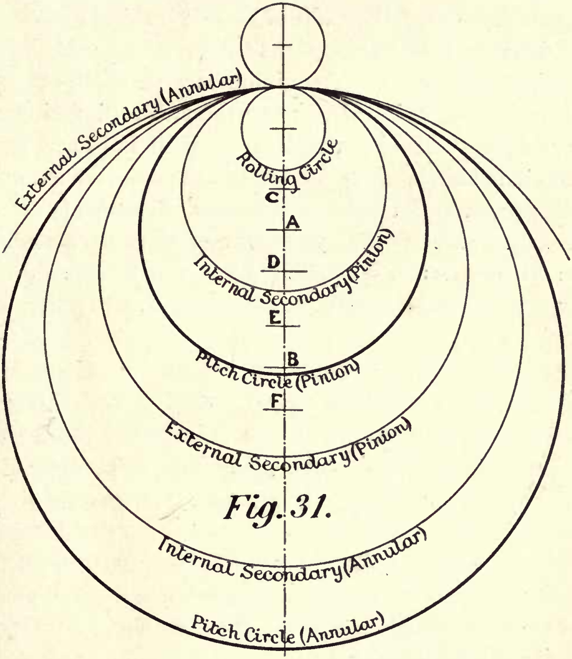


Fig. 31.

pinion will meet and have proper contact for driving, for they will have the same normal at their contact point and that normal will pass through the pitch point.

When this coincidence of secondary circles exists their centres D and E (Fig. 31) coincide, then the

centre distance AB of the wheel and pinion equals half the sum of the diameters of two rolling circles, for AD equals half the outer and EB half the inner; from which arises the rule, that for an annular wheel and pinion to gear together, both having cycloidal faces and flanks, the centre distance must not be less than the sum of the radii of the two rolling circles.

In Fig. 29 the proportions are such that secondary action does occur. The pinion is a standard one of 12-teeth and radial flanks and the two rolling circles are equal. The drawing is made with fair accuracy and small circles are placed round the points of secondary contact. There is no backlash and secondary contact theoretically changes to the opposite sides of the teeth on passing the pitch point, but practically it will be on one side only according to the direction of pressure.

If the wheel drives and rotation is clockwise, secondary action commences on the left at the intersection of the pinion's addendum circle and the circle of secondary contact, and continues to the pitch point. Before reaching that point ordinary or *primary* action begins and continues in the usual way, so that for the period of primary action from first contact to pitch point two points are in contact on one tooth, giving a wider distribution of pressure and affecting the resultant line of thrust.

29. Interference.—An inspection of the figures 29 and 31 will shew that if the annular wheel be made any smaller, its line of internal secondary action will fall within that of the external secondary action of the pinion, and for this to happen the teeth must penetrate each other, or what is known as *interference*

will take place and the gears will not turn; this defines the minimum limit of centre distance given in the rule in paragraph 28.

30. Influence of Rolling Circle on Limiting Velocity Ratio.—In Fig. 29 the ratio of wheel to pinion is 2 to 1; if less than this be required then the rolling circles must be reduced. For example, suppose a ratio of $1\frac{1}{2}$ to 1 be required: the difference in radii will be half the radius of the pinion, or in the figure, B will move to the place occupied by C, then since the centre distance AC must equal the sum of the radii of the two rolling circles, it may be divided into any two convenient parts that will give a proper form of tooth; if still smaller rolling circles be used secondary action will be avoided.

31. To Obtain a Small Velocity Ratio.—Another method of arranging for a small velocity ratio is to cut away the roots of one and the points of the other, then the difference in pitch diameters may be much reduced; no secondary action can then occur, but interference may and must be tested by drawing the path followed by the highest point of the pinion tooth as the pinion rolls within the wheel. Thus with a fine pitch the two pitch diameters may be brought very near to equality, which is natural as the smaller the pitch of the teeth more nearly does the wheel approach smoothness, in which limiting condition the annular wheel may be equal to the pinion and the two become a bush within a sleeve.

32. The Intermediate Rolling Circle.—Secondary action may be approached in another way.

In Fig. 24 circle centre D gave the same hypocycloid as circle centre A, from which may be gathered the

fact that two circles, the *sum* of whose diameters equals the diameter of the director circle, will each roll the same hypocycloid. A similar state holds for the epicycloid where, however, the *difference* of the diameters of the two rolling circles must equal the diameter of the director circle, and the larger one must roll concave to or contain the director.

In Fig. 32 PN is the pitch diameter of a pinion centre A and PM that of an annular wheel centre B touching at P. PQ and PT are the rolling circles centres R and S, PI = PN + PT = PM − PQ; the circle on PI is called the intermediate rolling circle. If the circles on PI and PT roll on the pitch circumference of the pinion they will both generate the same epicycloid, and if the circles on PI and PQ roll within the pitch circumference of the wheel they will generate the same hypocycloid. *p* is a chosen point on the intermediate rolling circle, and the curves *ab* and *cd* are parts of the resulting epicycloid and hypocycloid, and represent the face profiles of the teeth of pinion and wheel just as if they had been rolled by the S and R circles. All the properties concerning tangency, normal and path of contact of face and flank discussed in paragraphs 14 and 18, hold good, but in this case it is two faces that touch. First contact commences at P and ends at *l* where the addendum circle of the pinion cuts the intermediate rolling circle, the pinion driving clock-wise.

33. The Intermediate Rolling Circle and Secondary Contact.—The intermediate rolling circle is the same as the external secondary for the pinion and the internal secondary for the wheel when these latter two coincide, so that the path P*pl* is one of

secondary contact as illustrated in Fig. 29. The primary contact of course remains.

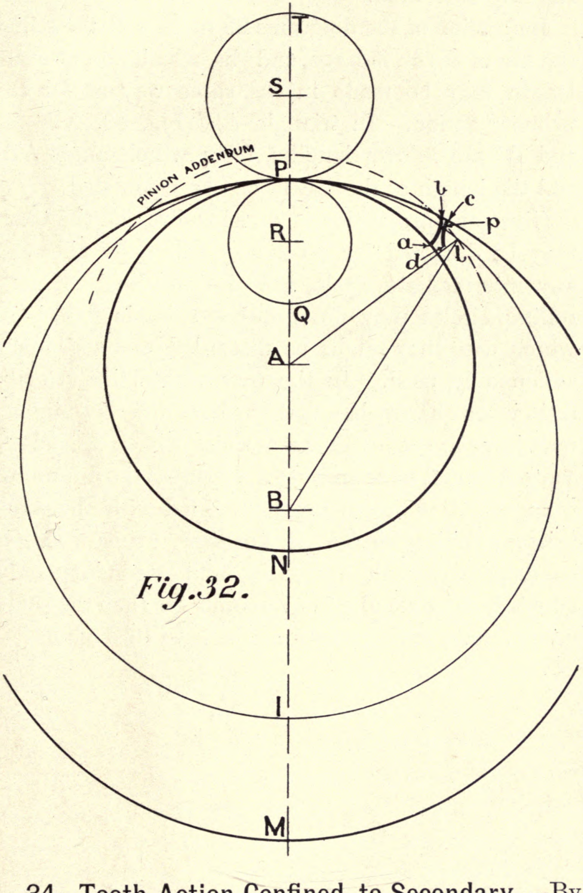


Fig.32.

34. Tooth Action Confined to Secondary.—By easing away the flanks of both wheel and pinion so that there is no contact there, action will be confined

to secondary only, and if the pinion is the driver it is wholly during recess—an advantage when quiet running is desired.

Inspection of the diagrams 29 or 32 will shew that the arc of action is large, and the calculation of exact length may be made in the same way as for the primary action. In triangle AB*l* Fig. 32, AB, A*l*, and B*l* are known lengths, from which angle AB*l* and the length of the arc P*l* can be calculated.

Given any proportion of wheel and pinion the teeth may be designed for secondary action by choosing any intermediate circle, but the resulting primary rolling circles may not produce suitable flanks, in which case they might be discarded and secondary action only used. In the case where the velocity ratio is too changeable, that is where different pinions may have to gear with the annular wheel, secondary contact cannot be arranged for, except at the minimum ratio, as all the teeth must be rolled with the same primary rolling circles; if, however, primary action be dispensed with altogether and secondary only adopted, the difficulty is overcome, for then the same intermediate rolling circle can be used throughout.

CHAPTER IV

THE SPUR WHEEL, INVOLUTE TEETH

35. The Involute Profile, Advantages.—The involute curve is better adapted for tooth profiles in that it has advantages not possessed by the cycloidal form. Wheels having involute teeth may have their centre distances altered slightly without spoiling the true rolling of the pitch circles, thus the teeth may be set more or less deeply into gear as required. The teeth can always be made strong at the root, so that this form can be used for a set of wheels ranging from the smallest to a rack. The tooth profile is one of single curvature. It also permits of a simpler design of gear cutting machine where no "former" is used but the tooth form is produced by the movements in the machine itself and not obtained by the copying principle.

36. The Base Circle.—In Fig. 33 XX and YY are parts of two pitch circles centres A and B, P is the pitch point, TT their common tangent at P, NN is a line making in the diagram 30° (for clearness of illustration) with TT, but in the design of actual teeth an angle from $14\frac{1}{2}°$ to $22\frac{1}{2}°$ is generally used. AD and BE are perpendiculars from A and B on to NN, RR and SS are circles centres A and B and radii AD and BE and touching NN at D and E respectively. These circles are called the *base circles*,

and from them the string is unwound to produce the involutes that form the profiles of the teeth upon XX and YY.

37. The Normal.—Let the line DE represent the visible portion of a taut string wrapped round the two base circles RR and SS in the manner indicated in the figure. If the string be cut at D and one end be unwound from RR, that end will trace the involute Da;

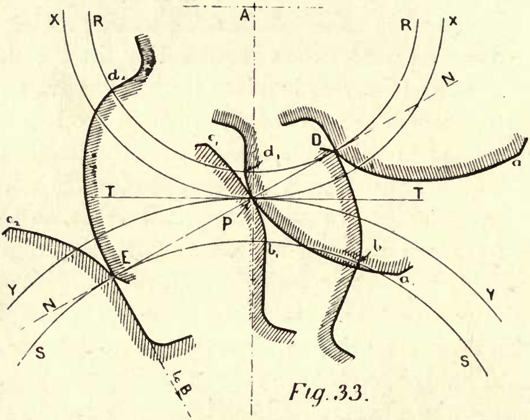


Fig. 33.

the other end if wound on to SS will trace Db. From the properties of the involute noted in paragraph 9 the normal to the two curves at D is the line DP.

If the string had been cut at P and the two free ends wound and unwound from their respective circles, the curves d_1a_1 and c_1b_1 would have resulted, with common normal ED running through their point of tangent contact P. Similarly any point might be chosen at which to cut the string, and the curves produced would have been identical in shape displaced only in position, and tangent con-

tact would occur at the cutting point of the string, and the normal in each case would have been the line DE passing through P.

38. Equal Pitch Line Velocity.—Instead of imagining different cuts suppose one pair of curves only, namely Da and Db, and that they turn about centres A and B respectively, retaining contact; the contact point traverses the line DE and after arriving at E is lost, the curves now separating.

The movement of Da to the position d_2E simultaneously with Db to C^2E causes SS to turn through the angle bBE and RR through the angle DAd_2, both equivalent to the winding or unwinding of a length DE; or in other words the base circles turn through the same circumferential distance. From the geometry of the figure the base circles are the same fraction of their respective pitch circles, viz:—

$\frac{AP}{AD} = \frac{BP}{BE} = \frac{\text{pitch circle}}{\text{base circle}}$; therefore the pitch circles will each turn through a circumferential distance equal to DE $\times \frac{AP}{AD}\left(\text{or} \times \frac{BP}{BE}\right)$. However small the movement, this equality still holds, and so the pitch circles roll without slip one upon the other.

With the above properties, the involute curve is admirably adapted for the formation of the working surfaces of wheel teeth. In the figure the curves are shaded to represent the important parts of a pair of teeth, and it will be seen that somewhere between D and E the tops of the teeth fall well below the base circle of the other wheel. The spaces then must be cut deep to clear. It must be noted, however, that there is no contact below the base circle.

A common practice is to cut down radially below the base circle and put a good fillet at the bottom corners, the depth being regulated by the addendum circle of the companion wheel. A stronger root may be obtained by making the flank below the base circle to just clear the corners of the tooth that gears with it; the path of the corners (called the line of least clearance) being obtained by rolling one wheel upon the other and plotting successive close positions of a tooth within a space.

39. Changing Centre Distance.—The further very valuable property that the centre distances of the wheels may vary somewhat without spoiling the accuracy of the tooth action is demonstrated by the aid of Fig. 34.

The shape of the teeth is not altered by changing the centre distance since the base circle and its resulting involute must remain the same in all positions when once made, but it is not self-evident that the tooth action is correct in a changed position. Separating or closing the centre alters the sizes of the pitch circles but not their ratio, it also varies the angle between NN and TT of Fig. 33.

In Fig. 34 centres A and B, pitch circles XX and YY, base circles RR and SS, and lines DE and TT are the same as in Fig. 33. The letters with the suffix 1 are for a new position of centre distance. D_1E_1 is drawn tangent to R_1R_1 and S_1S_1, and the diagram adjusted for the intersection of D_1E_1 and A_1B_1 to coincide with P; then by the properties of the similar triangles A_1D_1P and B_1E_1P,

$$\frac{A_1P}{B_1P} = \frac{A_1D_1}{B_1E_1} \text{ which equals } \frac{AD}{BE},$$

therefore A_1P and B_1P, the new pitch circle radii retain the old ratio AD to BE.

The argument applied to the initial position with respect to the contact and normal of the involutes from RR and SS holds equally for R_1R_1 and S_1S_1,

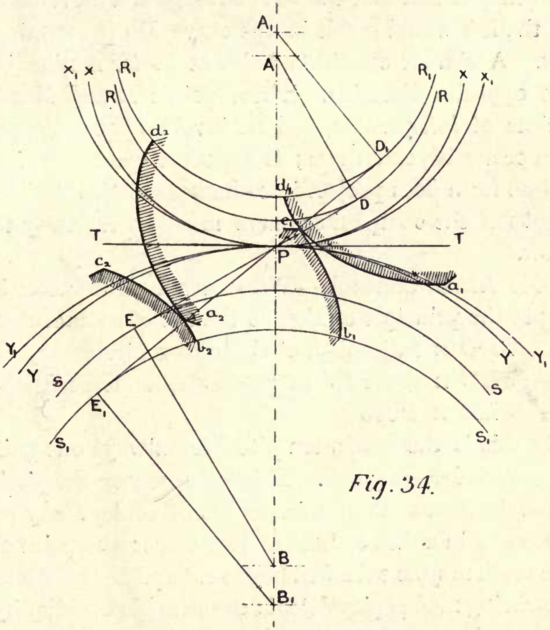


Fig. 34.

with the limitation that contact does not extend from D_1 to E_1 but only between c_1 and a_2; the involutes become separated beyond these points unless the original curves be extended in length. c_1 and a_2 are found by the intersection with D_1E_1 of circles with centres B_1 and A_1 and circumferences through the ends of the respective involutes (i.e. through the tops of the teeth).

40. Interference.—The wheels being designed for the position of Fig. 33, they cannot be brought closer together for this would cause the corner D to fall within the base circle RR, which would be equivalent to extending the involute beyond the point of tangency of DE and the base circle, and a movement to the left would result in the curve Db penetrating D*a*. A similar condition holds at E for motion in the opposite direction. These points D and E are points of *interference*, and no involute tooth action can occur beyond them; if for some reason the involute form does project into the base circle then the flanks of the companion curve must be cut away to clear.

41. Arc of Contact.—With teeth designed according to the principle of Fig. 33 the arc of contact

$= \text{ED} \times$ ratio pitch circle to base circle

$= \text{ED} \times \text{cosec } 15^\circ$ (if 15° be the angle used)

$= \text{ED} \times 1{\cdot}035$

and this is the maximum pitch for always one pair of teeth to have contact. If for some reason the pitch must be greater than this the arc of contact may be increased in either or both of two ways: the point of the tooth may be extended beyond D and E, the extension being made epicycloidal: or the angle of obliquity may be increased, thereby lengthening the distance DE and increasing the cosecant of the angle.

42. Minimum Number of Teeth.—In the case of two equal wheels, Fig. 35,

$\text{DE} = 2\text{DP} = 2\text{PE}$,

$\text{DP} =$ radius of pitch circle $\times \sin 15^\circ = r \times {\cdot}259$,

$\text{DE} = 2r \times {\cdot}259 = {\cdot}518r$,

arc of contact $= {\cdot}518r \times 1{\cdot}035 = {\cdot}536r$,

pitch circumference $= 2\pi r = 6{\cdot}28r$,
minimum number of teeth

$$= \text{pitch circumference} \div \text{arc of contact}$$
$$= 6{\cdot}28r \div {\cdot}536r = 11{\cdot}7 \text{ teeth};$$

or 12 as a practical limit.

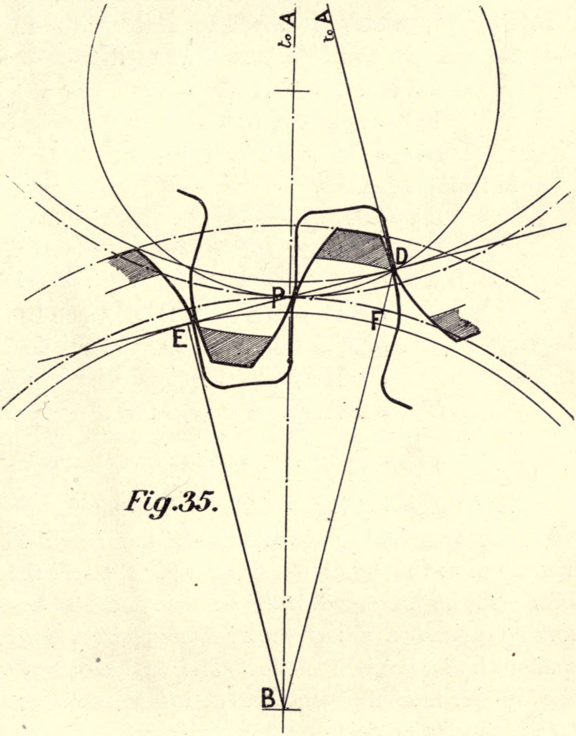


Fig. 35.

Neglecting the ratio of pitch circle to base circle—the angle turned through by the base circle during contact is 30° (twice the obliquity), then 360° ÷ 30° = 12 teeth as a minimum limit. This gives the

approximate rule, always on the safe side, that the *minimum number of involute teeth equals* 360° *divided by twice the angle of obliquity.*

EXAMPLE: given $22\frac{1}{2}°$ obliquity.

$$360° \div \text{twice } 22\tfrac{1}{2}° = 360 \div 45$$
$$= 8 \text{ teeth, minimum limit.}$$

In Fig. 35, which is drawn to scale with 15° obliquity and two equal pinions of 12 teeth,

$$\text{PA and PB} = \text{radius} = r,$$
$$\text{FD} = \text{height of tooth},$$
$$\text{BD} = \text{BF} + \text{FD} = \text{PB} + \text{FD};$$

in triangle BAD

$$\text{BA} = 2r,$$
$$\text{DA} = \text{PA}\cos 15° = \cdot 966r,$$
$$\text{BAD} = 15°,$$
$$\text{BD}^2 = \text{BA}^2 + \text{DA}^2 - 2\,\text{BA}\cdot\text{DA}\cos 15°,$$
$$= 4r^2 + \cdot 92r^2 - 2 \times 2r \times \cdot 966r \times \cdot 966$$
$$= 1\cdot 24r^2,$$
$$\text{BD} = 1\cdot 112r \;\therefore\; \text{FD} = \cdot 112r;$$
$$\text{pitch} = \frac{1}{12} \times 2\pi r = \cdot 523r$$
$$\therefore \text{height FD} = \cdot 112r \div \cdot 523r = \cdot 213 \text{ pitch.}$$

43. Epicycloidal Extension.—If the height is to conform to the usual standard, viz. ·3 pitch, the point can be extended with an epicycloidal face working against a radial flank; rolling circle being equal to half the pitch circle. The epicycloid will pick up the involute tangentially; for in the figure PDA is a right angle PD being a tangent at the extremity D of the radius DA of the base circle, then a circle on PA will pass through D, if now this circle describe an epicycloid from D DP will be its normal at D, and since DP is also normal to the

involute at D the two curves join tangentially; the combination is thus well suited for a tooth profile. The shaded part on the diagram is the epicycloidal extension; and with this extension the arc of contact is much greater than the circular pitch.

In order that the height of the teeth may be ·3 pitch without an epicycloidal extension the minimum number of teeth must be greater than 12.

It has just been shown that FD $= \cdot 112r$, if then FD is to be ·3 pitch, pitch $= \frac{10}{3} \times \cdot 112r = \cdot 373r$, and circumference $\div$ pitch $= 6{\cdot}28r \div \cdot 373r = 16{\cdot}8$ or 17 teeth as a practical limit.

As the wheels get larger, the pitch of teeth remaining constant, the arc of contact increases in the same way as with cycloidal teeth.

44. Annular Wheels.—Involute teeth may be fitted to annular wheels and their pinions, but there will be no secondary contact under any conditions. To find whether any interference will occur the pinion must be rolled within the wheel and the complete path of the corner of one tooth drawn; if it fouls a tooth of the wheel the design must be discarded and new dimensions chosen for the teeth or wheel diameters.

Fig. 36 illustrates a portion of a 15-teeth pinion with a 30-teeth wheel. The pitch lines are shown chain dotted and the base circles full lines, 15° obliquity is used and the height of the pinion teeth equals ·3 circular pitch. The path of contact aPED is tangent to the pinion base circle at E and the wheel base circle at D.

If the pinion be the driver and turn anti-clockwise contact commences at E and ends at a where the

addendum circle of the pinion cuts the path of contact. If the pinion drive clockwise contact commences at F and ends at *b*. The actual path of contact does not lie between the tangent points D and E, as with external wheels, but wholly on that side of both towards the pitch point, and is limited only by the working height of the pinion tooth.

The height of the teeth of the *annular wheel* is very small, being limited by the tangent point E, but the tooth is usually extended beyond E and the corners rounded to save abrasion there.

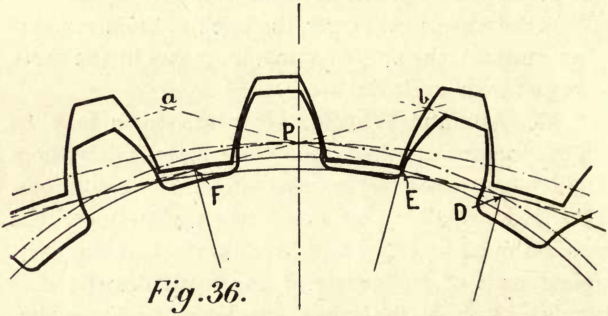


Fig. 36.

The arc of contact is just one and a half times the circular pitch; for a greater arc the points of the pinion teeth must be carried higher and the wheel spaces deepened to suit, the limit being reached when the two sides of the pinion teeth meet.

45. Involute Rack and Pinion.—The rack fitted with involute teeth is of interest owing to the sides of the teeth having no curvature. This straightness is due to the fact that the radius of curvature of a straight line is infinitely great. In Fig. 37 XX is the pitch line of the rack, YY that of the pinion,

XX is also the tangent at the pitch point, NN is at 15° to XX and BB is the base circle of the pinion and is tangent to NN at *t*.

Since the centre of curvature of the rack is at infinity that of its base circle is also at infinity, and the tangent point of NN with this base circle is at infinity in the direction NN to the right. If a point on NN within the picture turns about a point on NN at infinity it will describe an arc of infinite radius, which will be a straight line at right angles to NN.

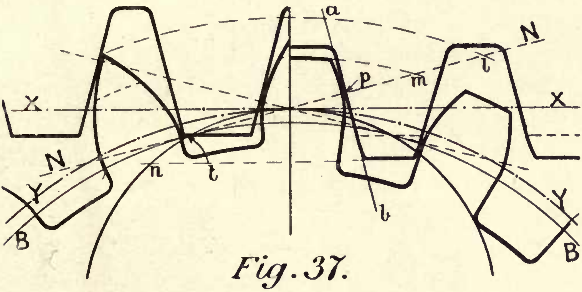


Fig. 37.

Selecting a point such as *p* and winding and unwinding to form the involute, the straight line *ab* is described at right angles to NN.

In drawing the involute teeth of the rack it is only necessary then to give them straight sides at 75° (or 90° – obliquity) to the pitch line and extending as far as conditions may need. Since one end of involute contact is at the tangent point *t* it is no use extending the straight sides beyond there, and the left side of the figure shows the rack teeth so limited, while on the right they are extended by cycloidal curves sufficient to give the tooth a height of ·3 circular pitch. The figure also shows the pinion teeth

in two different ways; on the right the height is ·3 circular pitch and on the left the profiles are carried up to meet in a point.

If the pinion drive clockwise, the contact for the left side extends from t to l, wholly involute contact, and for the right side, from n to t cycloidal contact and t to m involute contact.

CHAPTER V

OBLIQUITY OF ACTION, ETC.

46. Angle of Obliquity or Angle of Pressure.—The angle of obliquity is the angle which the normal at the point of contact makes with the common tangent at the pitch point; it is the angle which the coupler of Figs. 12 and 13 makes with the perpendicular to the line of fixed centres; it is the direction of the force being transmitted from one wheel to the other. If the angle be 0° then the normal coincides with the common tangent to the pitch circles, and the force is at right angles to the arm on which it acts (i.e. the line of centres), thus producing its maximum turning moment. If the angle be greater than 0° then the force may be considered to resolve itself into two components; one at right angles to the arm, the other along the line of centres thrusting the bearings apart and wasting energy in friction. This angle then should be kept as low as possible in order to obtain the best efficiency.

In the case of involute teeth the obliquity is constant, but in the case of cycloidal teeth it varies from a maximum at first contact to zero at the pitch point to another maximum at last contact, as can be seen on reference to Fig. 23. For heavy pressures a reasonable practical maximum is 25°, and 30° for moderate pressures; but as the angle changes more

rapidly near the ends of the path of contact than it does in the middle, the average is below half the maximum ; also two teeth are usually in gear for the greater part of the arc of contact which modifies the resultant direction of pressure at any moment.

With wheels of special design having only a few teeth and receiving very light pressures, the obliquity is disregarded entirely and smooth and accurate rotation only considered.

47. Unsymmetrical Teeth.—In wheels which always drive in one direction it is possible to keep the angle of obliquity very small by making the teeth unsymmetrical. The faces and flanks which receive the pressure may be formed by fairly large rolling circles producing undercut flanks, and to give the necessary strength at the root the backs of the teeth may be formed from involutes or by small rolling circles. In Fig. 38 the rolling circles for the working surfaces are three quarters of the pitch circles, and the backs are involutes with 45° obliquity. The maximum obliquities of the working surfaces are 16° 42′ on the left and 10° 48′ on the right of the pitch point.

48. Sliding Action.—On reference to Fig. 23 it will be seen that the curves which form the faces of the teeth are longer than those of the flanks on which they work ; aP is longer than dP, and kP than bP ; there is then an amount of sliding equal to the difference of length in each case. During approach this sliding is supposed to be of the nature of pushing and during recess of trailing, so that the motion during recess is smoother ; for this reason, in cases where smoothness of motion is of great importance, the arc

of recess only is used, the face and flank of approach being removed entirely.

Involute teeth may be examined in the same way by the aid of Fig. 33.

49. Conjugate Teeth.—The profiles of the teeth of wheels need not be confined to the cycloidal or involute curves, it is possible to select any shape for

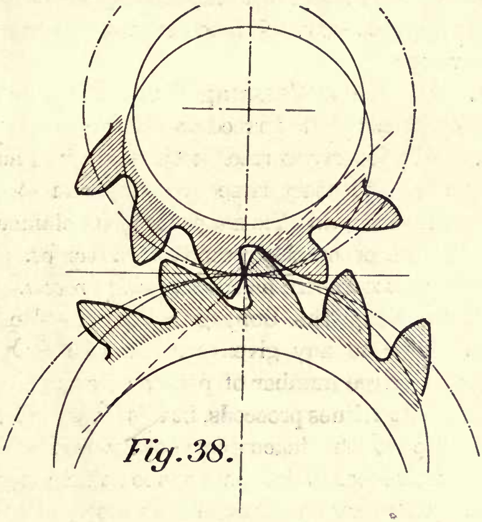


Fig. 38.

the tooth of one wheel and yet find a companion to gear with it theoretically, and if the selected shape is fairly uniform in thickness, and not too long, a companion tooth can be found to gear with it practically.

Assuming no backlash, the process of finding the shape of the companion is by rolling the pitch line of the original wheel, with the chosen form attached in proper position, upon the pitch line of the second wheel, and obtaining the envelope of the chosen form.

This envelope will be the companion space on the second wheel.

50. The Envelope.—If a line, straight or curved, move about within a plane and a second line be found that is tangent to the moving one in all its positions, the second is called the *envelope* of the first. The reverse motion holds true, if the envelope found be made the moving line and caused to travel in an exactly opposite way, then the original line becomes the envelope.

51. All Truly Meshing Teeth Conjugate.—Teeth of wheels thus formed are known as *conjugate teeth.* All properly formed teeth whether cycloidal, involute or any other shape are conjugate to those they mesh with, and the shapes may be obtained one from the other by the method of envelopes. This method is also known as the *moulding process.*

52. Finding the Conjugate Curve.—The conjugate tooth to any given form may be found by drawing a large number of positions for the tooth as rolling of pitch lines proceeds, but this is very laborious and subject to the inaccuracies of drawing. With a fairly simple mechanical contrivance a much more accurate profile may be obtained with ease and speed.

In Fig. 39 A and B are two pins, fixed to any convenient base, round which can rotate two discs C and D. Each disc carries a graduated scale E and F and is clamped and indexed at G and H respectively. To C is attached the template of a tooth and to D is fixed a sheet of thin metal or cardboard which is supported underneath so as to receive the pressure of a scriber or pencil. The size of the sheet on D and the distance of the template from centre A are

made to suit the sizes and velocity ratio of the wheels. If one of the pins, say B, be made to work in a slot, the centre distance AB can be adjusted and the apparatus is more useful.

It is not necessary to know the pitch lines, shown chain-dotted; only the velocity ratio is needed. Having the velocity ratio, a line is drawn close up to and all round the template, the discs are unclamped and each turned a small amount in proper direction

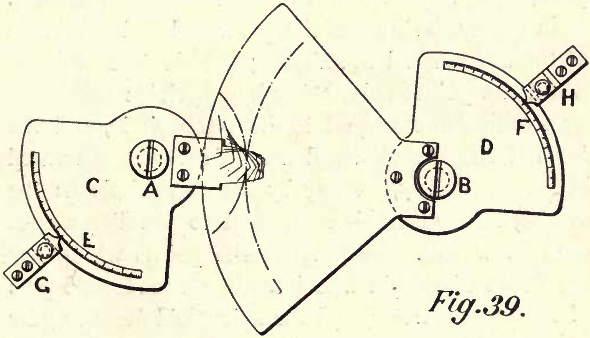


Fig. 39.

and proportion. With a 2 : 1 ratio as in the figure, A is turned through two degrees and B through one degree in the opposite direction, both again clamped and a line drawn round the template. Repeating until sufficient marks are obtained the envelope of the template may be drawn in on one side; going back to the starting point and operating in the opposite direction, the envelope on the other side may be found, and thereby the whole space. If the different positions are taken close enough together there is no need to draw a separate tangent curve for the envelope, the shape will be quite distinct without it.

CHAPTER VI

PIN GEARING

53. Pin Gearing.—A very useful form of wheel is one in which the teeth are merely pins. Such an arrangement can be made to work with accuracy practically as perfect as the teeth already described.

In theory the faces of the pin wheel and the flanks of the other wheel are absent; the flanks of the pin wheel are points only, being hypocycloids generated by a rolling circle equal to the pitch circle, and the faces of the second wheel are epicycloids generated in the usual way, namely by the same circle as the mating flanks (in this case equal to the diameter of the pin wheel). In practice the points which are the flanks of the pin wheel teeth are made into pins of sensible diameter, and the epicycloid faces to gear with them are cut away by an amount equal to the radius of the pins.

54. Theoretical Form.—In Fig. 40, wheel A has 8 points equally spaced upon its circumference, wheel B has teeth with epicycloidal faces formed by the rolling of A on the pitch circle of B. Supposing the points on A to be rigid wires of no thickness, A might drive B or B might drive A.

55. Practical Form.—Fig. 41 shows the practical case of the wheel being built up of a series of pins (usually rivetted to two end discs and supported on a spindle, in which form it is called a lantern pinion) and the mating wheel teeth profiles being lines paral-

lel to epicycloids and distant from them by the radius of the pins.

56. Pin Wheel and Rack.—Fig. 42 shows the application to a rack; here the theoretical curves

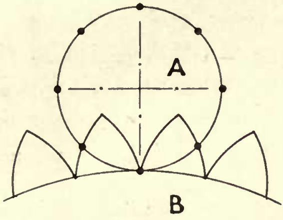


Fig. 40.

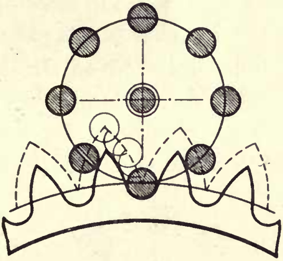


Fig. 41.

for the rack teeth are simple cycloids generated by the rolling of the pitch circle of the pin wheel upon the pitch line of the rack. In Fig. 43 the rack is driven and carries the pins; the curves for the pinion

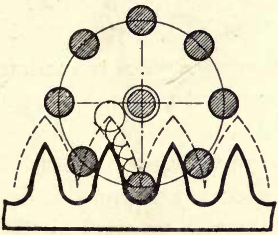


Fig. 42.

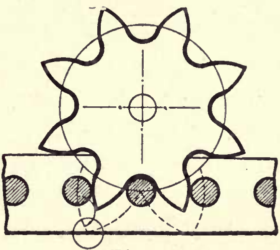


Fig. 43.

teeth are parallels to involutes generated by rolling the rack pitch line upon that of the pinion.

57. Internal Gearing.—Figs. 44 and 45 illustrate the two cases of internal gearing. The principles of construction are the same as for external wheels with the exception that the epicycloid becomes a

hypocycloid. In Fig. 44 the teeth of the wheel are not made pointed, the height being made consistent with a sufficient arc of contact. In Fig. 45 where the annular wheel carries the pins, the rolling circle used to obtain the faces of the pinion teeth is equal to the diameter of the pin wheel.

58. Direction of Driving.—In pin gearing it is usual to drive the pin wheel. When that is the case action is wholly during recess and the movement is very smooth and silent. The pin wheel will act as

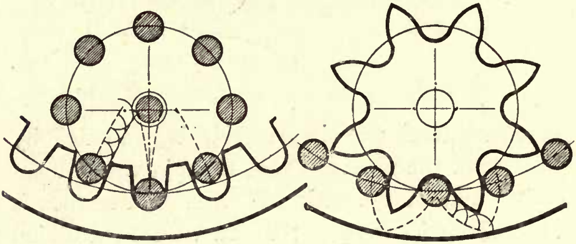


Fig. 44. *Fig. 45.*

driver with equal theoretical accuracy, but the action is then wholly during approach and the smoothness of movement is sacrificed.

59. The Inaccuracy of Pin Gearing.—There is a defect in the theoretical accuracy of pin gearing. The usual assumption is that the parallel to the cycloid or epicycloid is one smooth curve, and that when the centre of the pin is at the pitch point the lowest working spot of the tooth face is just tangent to the pin and the common normal passes through the pitch point.

Close investigation shows that the parallel line is not a continuous smooth curve. In Fig. 46 ABCD

is a cycloid parallel to which is to be drawn a curve. Many normals are set out and equal distances marked off upon each and the curve *abcd* drawn through.

It will be noticed that coming downward and nearing the end A of the cycloid, the parallel curve suddenly takes a new path; the reversal or cusp is seen at *b* and is on the normal at B, the short length *ba* is parallel to the portion BA.

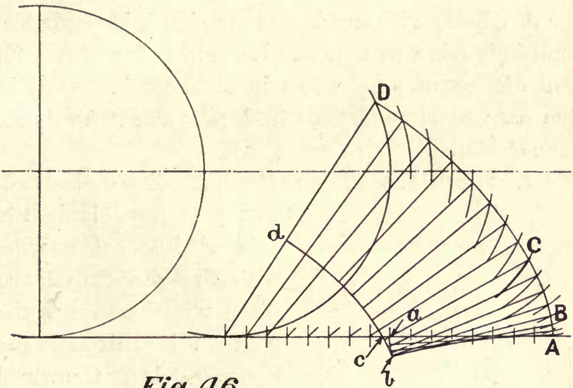


Fig. 46.

Now it is quite evident that a tooth face cannot in its practical form contain the whole curve *dcba*, the piece *ba* must be taken away; and this is the defect in the practical tooth. The portion BA of the cycloid is out of theoretical action and the part *ba* of the parallel curve is lost to practical action. First contact does not occur until point B comes into play, that is when *b* is tangent to the pin and the normal B*b* passes through the pitch point; the centre of the pin will then have passed the pitch point so that the arc of action is shortened.

The case of the epicycloid is worse than that of the cycloid, the part *ba* being longer.

60. The Defect Negligible.—The dimensions chosen to draw Fig. 46 exaggerate the defect immensely; there the parallel curve is distant from the cycloid a distance equal to the radius of the rolling circle, a condition that would never occur in practice. As the ratio of the rolling circle to the pin gets greater the defect becomes less, and in the case of small wheels with much smaller pins it is negligible, sufficient evidence of which is found in the wonderful smoothness of pin wheels in clock-work. Only in the case of large wheels and pins need the defect be considered.

61. The Case of no Defect.—Theoretical accuracy is possible in the case of internal gearing with a ratio of 2 to 1, the small wheel carrying the pins. With this ratio the hypocycloids for the wheel teeth are straight lines and parallels to these will also be straight lines, so that there is no cusp and no defect. The straight sides of the teeth lend themselves to the peculiar construction seen in Fig. 47; the spaces between the teeth are formed into proper grooves in which a block can slide and connexion with the pinion is made by pins in the blocks. The body of the pinion wheel may be a plain disc or may be constructed with arms as illustrated.

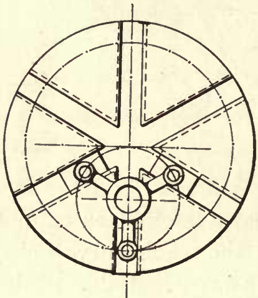


Fig. 47.

CHAPTER VII

NON-CIRCULAR WHEELS

62. Non-circular Wheels.—The wheels thus far considered have all had circular pitch lines and the ratio of the angular velocities of the shafts connected has been constant.

It sometimes happens that two parallel shafts are required to be connected and that while the angular velocity of one, the driver, remains constant, that of the follower is to vary between a maximum and a minimum, and perhaps in addition to have some particular angular velocity constant for a definite angular displacement each revolution; or it may happen that actual angular velocity is unimportant but that the second shaft is to be subjected to a varying torque (turning moment). A good example of the former case is a quick return motion for a machine carrying a reciprocating tool, and of the latter a steering gear for a large vessel. These two results can be obtained by means of toothed gearing, but the pitch lines will not be circular, or at least not concentric with the axes of rotation.

Any form of pitch line may be assumed together with its axis of rotation and a second axis chosen, and it is possible to find by construction a second pitch line which will roll without slipping upon the first (or arbitrarily chosen one) as each rotates on its

axis. The assumed pitch line may be a closed curve but it does not follow that the second (or derived) one will also be a closed curve about its chosen axis, but by the method of trial and error a position can be found for the second axis about which the derived pitch line is closed.

63. The Derived Curve.—Fig. 48 illustrates the process of finding the derived curve. XX is the arbitrarily chosen pitch line and O_1 its axis of rotation, O_2 is the second axis, P is the intersection of the centre line with the chosen curve and is the pitch point for the position drawn. As XX rotates, the points A, B and C will each in turn arrive at the line of centres O_1O_2, circles about centre O_1 and through the points cut O_1O_2 in a, b and c. About O_2 as centre indefinite arcs of radius O_2a, O_2b and O_2c are drawn, and with distance PA in compass and centre P arc aA_1 is cut in A_1, with AB in compass and centre A_1 arc bB_1 is cut in B_1; similarly C_1 is found, and a smooth curve drawn through $PA_1B_1C_1$. Since $PA_1 = PA$, $A_1B_1 = AB$ and $B_1C_1 = BC$, the whole arc $PA_1B_1C_1$ will equal PABC in length, and by construction $O_1O_2 = O_1A + O_2A_1 = O_1B + O_2B_1 = O_1C + O_2C_1$; then the two curves may rotate about their axes and roll upon each other without slipping and the contact point will always be upon the line of centres.

Any number of points may be constructed upon this principle and the closer the points are taken the more accurate, in theory, will the derived curve be.

The angular velocity ratio of the two pitch lines varies every instant during rotation since the pitch point travels along O_1O_2.

64. Quick Return Motion.—Fig. 49 shows the form of two pitch lines which give a constant angular velocity ratio for a portion of a revolution and a variable one for the remainder. The part ABC of the lower wheel, centre O_2, is a circular arc of 240°, and gears with the part DEF of the upper wheel which is circular for 180°; radius O_2A is $\frac{2}{3}$ of O_1D, so that

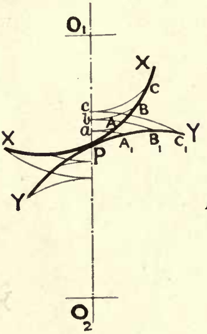


Fig. 48.

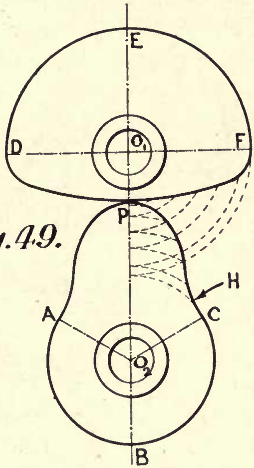


Fig. 49.

the velocity ratio during the rolling of these arcs is 3 to 2. For the other portions either DPF or APC may be chosen and the other derived from it, three or four trials being probably necessary before the two curves are found of exactly the proper length. Concave parts such as H should not be too small a radius or the teeth when fitted may interfere.

In the position drawn the velocity ratio is the least, viz. lower to upper is as O_1P to O_2P which is about 1 to 3. Assuming the lower one to rotate uniformly

in a clockwise direction, and starting with A and D in contact, the upper one will rotate uniformly for half a revolution with two-thirds the angular velocity of the driver, and for the other half revolution will increase to three times and again diminish to two-thirds. A reciprocating tool connected to the upper shaft could be made to make its forward or working stroke during the period of uniform rotation and its quick return during the variable period.

65. Variable Turning Moment.—An example

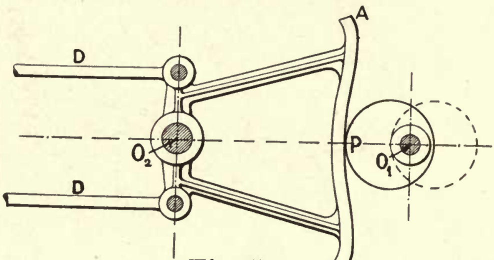


Fig. 50.

where the actual velocity may be disregarded but where the leverage or turning moment is of prime importance is seen in Fig. 50 which illustrates Harfield's Steering Gear. Here the four-sided piece rotating about centre O_2 is attached by drag links DD to the rudder arm, and the part APB is fitted with teeth which gear with the pinion set eccentrically on centre O_1. As the helm is put over the resistance to movement is greater, but at the same time the lever radius of the pinion gets less and that of the quadrant piece greater; thus the mechanical advantage of the combination increases as the resistance increases.

66. Elliptic Wheels.—Non-circular wheels with

their pitch lines formed from regular curves are found in elliptic wheels.

If two equal ellipses are each mounted so as to rotate about a focus and the distance between the axes of rotation equals the major axis of the ellipse, the two curves will roll without slipping, each making a complete revolution in the same time, the angular

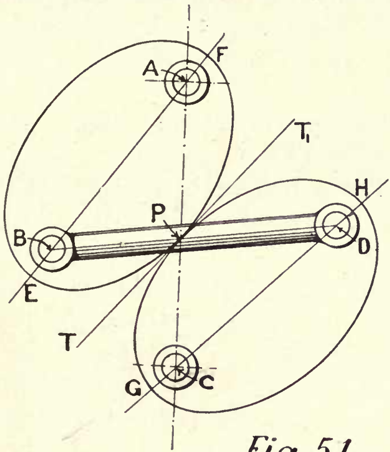


Fig. 51.

velocity ratio varying between a maximum and a minimum which each occur once during a rotation.

67. The Ellipse, some Properties.—An ellipse is the locus of a point moving in a plane such that the sum of the distances of the moving point from two fixed points in that plane is constant. The fixed points are called the foci, and the lines joining the moving point in any position (i.e. any point on the ellipse) to the foci are called the focal distances. Referring to the upper curve of Fig. 51, A

and B are the foci or fixed points, and the moving point travels the path drawn, which is the ellipse. Choosing point P on the curve. PA and PB are its focal distances and their sum by definition given above equals the sum of EB and EA or FA and FB, from which it may be deduced that EB = FA, therefore the sum of the focal distances = EF; the line EF is called the major axis. The line bisecting EF at right angles and limited by the curve is called the minor axis. The tangent, as TT_1, at any point bisects the exterior angle between the focal distances, and in the figure it is shown bisecting the angle between AP and BP.

68. Ellipses Rolling in Contact.—The foci of the upper ellipse are A and B, and those of the lower one C and D, the curves are equal and are mounted so as to rotate one about A the other about C, AC equals the major axis EF or GH, and P is their point of contact which is on the line AC.

These conditions can be shown to be true by the following argument. Selecting a point P say on the upper ellipse and a similar point on the lower one and placing the selected points in contact as shown, the angles which the common tangent TT_1 makes with the four focal distances are all equal, viz. BPT = APT_1 = CPT = DPT_1, and since CPT = APT_1, AP and PC are in one straight line, also since PB = PC, being similar lines though on opposite sides, the distance AP + PC = major axis, therefore AC = major axis and contact point is upon it. Similarly B, P, and D are in one straight line and BD = major axis and crosses at P. If any other pair of similar points had been chosen for the contact the same

reasoning would apply, so that contact being always between similar points oppositely placed one on each ellipse the two curves will roll upon each other. If A and C be made axes of rotation the ellipses will roll as they rotate, each making a complete revolution in the same time, and contact will be always on the line of centres.

Since BD is constant and equals the major axis, the points B and D may be connected by a link and the rotation of the ellipses proceed without fear of separation.

During the rotation E and G arrive at the line of centres together, then one of the limits of velocity ratio is reached, also F and H will come together on AC and then the other limit is reached.

69. Logarithmic Wheels.—Another well-known curve with useful geometric properties which lends itself to the formation of pitch lines of non-circular wheels is the logarithmic or equiangular spiral; but this curve being an unclosed one will not allow of continuous rotation in one direction.

70. The Logarithmic Spiral, some Properties.—A logarithmic spiral is a curve which consists of an indefinite number of convolutions round a point called its pole, the radius of the curve ranging from point to point in such a manner that while the angle between the radii increases in an arithmetical ratio the length of the radius increases in a geometrical ratio.

In Fig. 52, suppose the spiral ABC to I given with the pole O; if AOB be any chosen angle and AOC equals twice AOB, then if OA be taken as unity OC equals the square of OB, if AOD be three times

AOB then OD equals the cube of OB, and so on. Expressed algebraically

$$OC \times OA = OB^2 \text{ and } OD \times OA = OB^3,$$

and so on. Equations such as these hold for any position, thus:—

$$OF \times OH = OG^2 \text{ and } OE \times OH = OF^3;$$

or the property may be expressed in the following words. The radius which bisects the angle between two radii is the mean proportional between those two radii.

71. Drawing the Logarithmic Spiral.—The curve may be drawn by choosing any pole and plotting a series of equal angles, choosing the first and second radii and calling the first one unity, the third, fourth, etc., can be marked off equal to the square, cube, etc., of the second one. Or by another method, the limiting radii of the curve may be first chosen together with the angle they include, as OA and OI including 360°, and bisecting the angle OAI in OE make $OE = \sqrt{OA \times OI}$; bisecting IOE in OG, $OG = \sqrt{OE \times OI}$, similarly $OC = \sqrt{OA \times OE}$; and so on a series of points may be obtained.

72. Other Properties.—Other important properties are: the angle between the tangent and the radius at the tangent point is constant; also if a given length be measured on the curve anywhere, the difference in length of the two radii at its extremities is constant, thus in the figure arc ACE = arc FGH, then $OE - OA = OH - OF$. The length of the spiral is not limited either end, for both methods of construction might be carried to any extent in both directions.

The fact that equal lengths of arc give equal differences of radii at once gives the curve the required property of rolling and rotating with another equal curve, and the capacity to produce any change of velocity ratio that may be desired during a whole or partial revolution.

73. Logarithmic Spirals in Rolling Contact.—Fig. 52 shows two equal logarithmic spirals set so as to rotate about their poles O and P. The centre distance is such that as rotation proceeds I will come to *a* and *i* to A simultaneously, and continuous rotation in one direction might go on if the sudden change of velocity ratio that would occur at the moment *i* came into contact with A were permitted.

CHAPTER VIII

LOBED WHEELS

74. Lobed Wheels, Logarithmic.—By using 180° of the spiral in duplicate on each wheel, continuous rotation without sudden change of velocity ratio is obtainable; in Fig. 53 the 180° from A to E of Fig. 52 is used in this manner. The two wheels there are equal in every respect and will gear together correctly if their axes of rotation are set as illustrated.

Fig. 54 shows an arrangement where the 90° of arc, C to E of Fig. 52, is used four times on each of a pair of wheels; this will give two maxima and two minima velocity ratios during one rotation.

In Fig. 55 the wheel on the right uses the 180°, E to A, and the one on the left the 90° F to H of Fig. 52, these arcs being the same length, and the resulting total rotations are 2 to 1 in the same time; the actual velocity ratio at any instant will depend upon the points in contact at that moment.

Wheels of the type of Figs. 53 to 55 are known as *lobed wheels;* Fig. 53 being a pair of unilobes, Fig. 54 a pair of bilobes, and Fig. 55 a unilobe working a bilobe. It must be noted that any unilobe will not work with any bilobe although obtained from the same spiral, but the length of arc and included angle must be such as to permit of the continuous rotation.

Trilobes and other multilobed wheels can be ob-

tained in the same manner as unilobe and bilobe. A length of arc equal to ACE and including 60° would give one-sixth of the pitch line of a trilobe to work

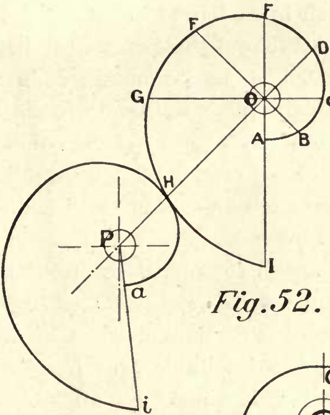


Fig. 52.

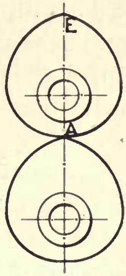


Fig. 53.

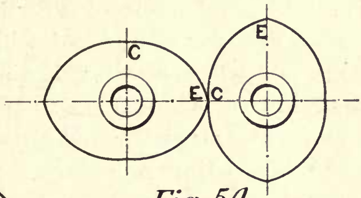


Fig. 54.

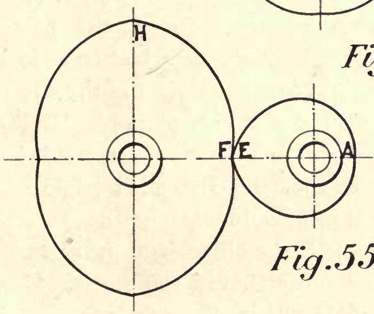


Fig. 55.

with the unilobe or the bilobe of Fig. 55, and in like manner any other multilobe could be obtained to work with them; and thus an interchangeable set may be produced.

75. Lobed Wheels, Elliptic.—The ellipse is a curve from which may be derived other curves suitable for the pitch lines of lobed wheels. Two elliptic wheels each rotating upon a focus as already discussed constitute a pair of unilobes; but if the same ellipses were set to rotate about their centres and the centre distance adjusted so as to obtain contact, although one wheel might push the other round, if smooth, they would not have proper rolling contact, the contact point would not always be upon the line of fixed centres, so that such an arrangement cannot be classed as a pair of bilobes.

To derive suitable curves for multilobes from the ellipse, a process known as the *reduction of angles* is used. The whole construction consists of two parts: first the selection of suitable arcs from two equal ellipses; and then reduction of the angles subtended by these arcs at the fixed foci. Fig. 56 illustrates a case for a bilobe and a trilobe to work together.

76. Selecting the Arcs.—Given the pair of ellipses in contact at E and axes of rotation at foci A and C, the lower axis is to make three rotations to the upper one's two. Find a length of arc as ENK on the lower ellipse equal to a length of arc EML on the upper and such that the subtended angle ECK is $\frac{3}{2}$ times EAL; if this be done, while the arc ENK rolls upon EML the lower ellipse will have turned through an angle $\frac{3}{2}$ times that of the upper. These arcs ENK and NML will be the portions used for the derivation of the lobes.

The difficulty lies in finding the point K and its similar point L so that the arcs subtend the proportionate angles at their respective foci of rotation.

Points K and L are found by a method of loci, thus: from the focus C is set out an angle ECa and from D is set out EDa' = $\frac{2}{3}$ ECa, and similarly are set out EDb' = $\frac{2}{3}$ ECb; EDa' = 100°, ECa = 150°; EDb' = 105°, ECb = 157$\frac{1}{2}$°, and so on, and a smooth curve

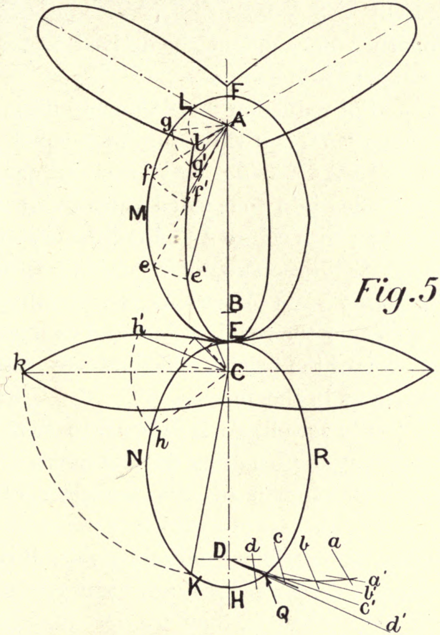


Fig. 56.

is drawn through the intersections of the pairs. The curve so drawn is the locus of all points from which if lines be drawn to C and D the exterior angle at C is $\frac{3}{2}$ times the interior angle at D. The intersection Q of this locus with the ellipse gives the required point. Q is transferred across to K and again

up to L; EAL = EDQ, ECK = ECQ, and arc EML = ERQ = ENK.

77. Reduction of Angles.—With this preliminary operation completed, the angle EAL is to be reduced to 60° and ECK to 90°.

Since $\frac{\text{EAL}}{\text{ECK}} = \frac{2}{3} = \frac{60^\circ}{90^\circ}$ the reduction will be the same proportion in each and the derived curves will be the same length in each.

The method of reduction in the upper ellipse is as follows: Angle EAL is divided into four equal parts by the lines A*e*, A*f* and A*g*, EA*l* is set out equal to 60° and is divided into four equal parts by the lines A*e′*, A*f′* and A*g′*; with centre A and radius A*e* an arc is drawn to cut A*e′* in *e′* and similarly A*f* is transferred to A*f′*, A*g* to A*g′*, and AL to A*l*; a smooth curve through *lg′f′e′*E is the required one and is one-sixth of the perimeter of the trilobe, and the three lobes can now be drawn as in the figure.

ECK of the lower ellipse is reduced to EC*k* (90°) in the same way, the resulting derived curve is E*h′k* which forms one-fourth of the perimeter of the bilobe.

Although it is possible from any given ellipse to derive a pair of lobed wheels of any required number of lobes to work together, they are not interchangeable; each pair must be separately derived, and no single one will work with any other than the mate with which it was derived.

78. Teeth for Non-circular Wheels.—The tooth profiles of non-circular wheels may be formed in the same manner as for circular wheels, rolling circles being used and the faces and flanks being of the

cycloidal form. Since the pitch curves roll upon each other and the pitch point is always on the line of centres, a circle will roll with and upon both pitch lines, and a tracing point will mark out the cycloidal curves; and the normals to the curves at the contact point will pass through the pitch point precisely as in the case of circular wheels.

The choice of rolling circle will be regulated by the greatest curvature of the pitch line; if the teeth there have radial flanks then the radius of the rolling circle will be half the least radius of curvature.

Usually it is possible to find circular arcs which practically coincide for some distance with the non-circular pitch lines, and the whole pitch line can be traversed by a few such arcs, in which case the problem of the teeth profiles is the same as for circular wheels only it must be handled in pieces. Of course the perimeter of the pitch line of all wheels, whatever the shape, must be capable of exact division by the circumferential pitch of the teeth.

In cases where there is a change, especially if it be an abrupt one, in the outline of the wheels care must be taken that a face of a tooth falls at that spot, in order that the action between the teeth may be continuous and the rotation go forward without check.

79. Obliquity of Action.—The obliquity of action generally is excessive, for the common tangent at the pitch point is usually already oblique and the obliquity due to the tooth profile is additional.

In Fig. 57 AB and CD are portions of two non-circular pitch lines and TT the common tangent at P, RR is at right angles to the line of centres GH,

PLE is the rolling circle and L is the initial contact.

The common tangent has obliquity ϕ = angle TPR and at first contact there is added to this θ = LPT, making total obliquity $\phi + \theta$ = LPR. The obliquity at last contact is obtained similarly but by the lower circle. 50° should be looked upon as the

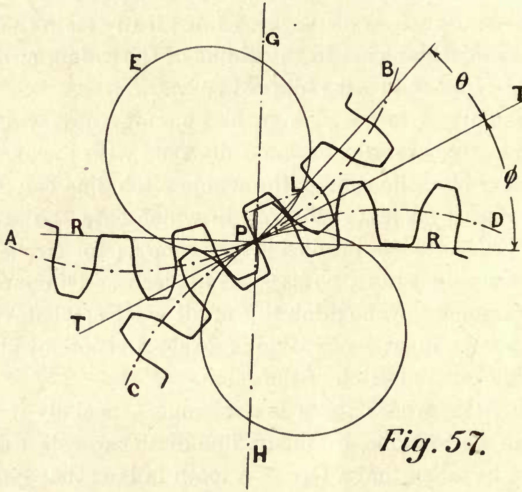


Fig. 57.

maximum total obliquity, and the endeavour of the designer should be not to exceed it.

Maximum obliquity will in general differ for each tooth on account of the varying obliquity of the common tangent itself and the consequent direction of the rolling circle which stands upon it at P in its fundamental position.

In the figure initial contact is approximately at L, L being found by the intersection of a circle about the

centre of rotation and through the top of the tooth in gear. This is not exact for when the tooth in question nears the position of initial or final contact the rolling circle may have altered its fundamental position to suit some change in direction of the common tangent. Exactness could be obtained only by trial and error.

Logarithmic pitch lines in contact will have a common tangent making a constant angle with the line of centres, and the maximum obliquity can be found by the direct graphical process as the fundamental position of the rolling circle under these circumstances does not change.

CHAPTER IX

HELICAL WHEELS

80. Helical Wheels.—Ordinary well-made spur teeth have contact in a line across their whole breadth, and this line comes into action all at once as the tooth falls into gear. With well-cut gears with the points of the teeth eased off and running at low pitch line velocities this contact may be picked up with little or no shock, but with teeth not so excellently made and in all cases of high velocity there is more or less shock at this moment, causing noise and vibration.

In the early days of mechanical science, long before cut gears, Dr. Hooke (1635-1703) invented what are known as "stepped wheels" for overcoming some of the shock experienced with toothed gearing. The wheel was made in the form of a number of narrow wheels or plates placed side by side with the teeth of each displaced a small amount circumferentially from its neighbour, as in Fig. 58, so that as contact came on it was for a portion of the whole width only; the combination has the effect of multiplying the number of teeth by the number of sections, while retaining the original pitch.

Obviously by increasing the number of sections the shock at contact is still further reduced, but the width of a single section will be limited by the strength

of the material and the number of sections by the difficulty of accurate production, so that further improvement is soon stopped. However, by making the width of a section infinitely small and the number of sections infinitely great, and making the circumferential advance of each section equal and in the same direction, the appearance and actual state of the wheel is as if it were twisted or as if the teeth were set obliquely across the wheel; but the teeth

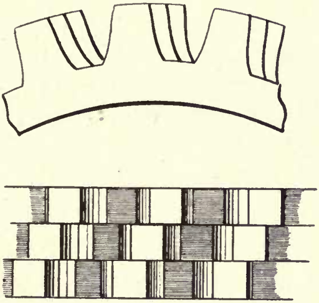


Fig. 58.

lying close to the cylindrical pitch surface they are portions of helices, and the wheel is known as a *helical wheel* or wheel with helical teeth, and still another name is *Hooke's spiral gearing.*

Since the teeth are oblique there will be end thrust, for the pressure being at right angles to the tooth surface it is not in a plane at right angles to the axis of rotation. To overcome the end thrust such wheels are made with teeth inclined in both directions forming both right and left-handed helices, as in Fig. 59; the end thrust is then automatically neutralized.

81. Tooth Contact.—Helical wheels are also made with two and three reversals. Taking any plane section at right angles to the axis the *contact* of the teeth is the same as for simple spur gearing, and in designing the teeth the profiles must be so treated, although the direction of resultant pressure is inclined to the plane of section.

If the wheels be single helical, contact commences at the end of the tooth that first comes within the arc of contact, and gradually increases in length until when the whole tooth is within the arc it extends right across, though not in a straight line, and continues in this condition until the tooth begins to leave the arc, when it diminishes till it disappears at the opposite end of the tooth to that at which it began. From this it can be seen that if the axial advance of the helix per width of wheel be equal to the circular pitch, and the circular pitch less than the arc of contact, at least two pairs of teeth will be in gear at any instant.

If the wheel be double or treble helical each part may be considered independently in the same manner as the single wheel.

Fig. 59 shows an equal pair of wheels, and it can there be seen that a right-handed helix on one works with a left-handed helix on the other. In equal or unequal pairs the circumferential advance per width of tooth must be the same on each wheel, hence the angle or inclination of the helix to the axis of the pitch cylinder must be the same on each.

82. The Helix Angle.—There is nothing to rigidly fix the inclination of the helix to the axis of the wheel. The practical considerations which influence

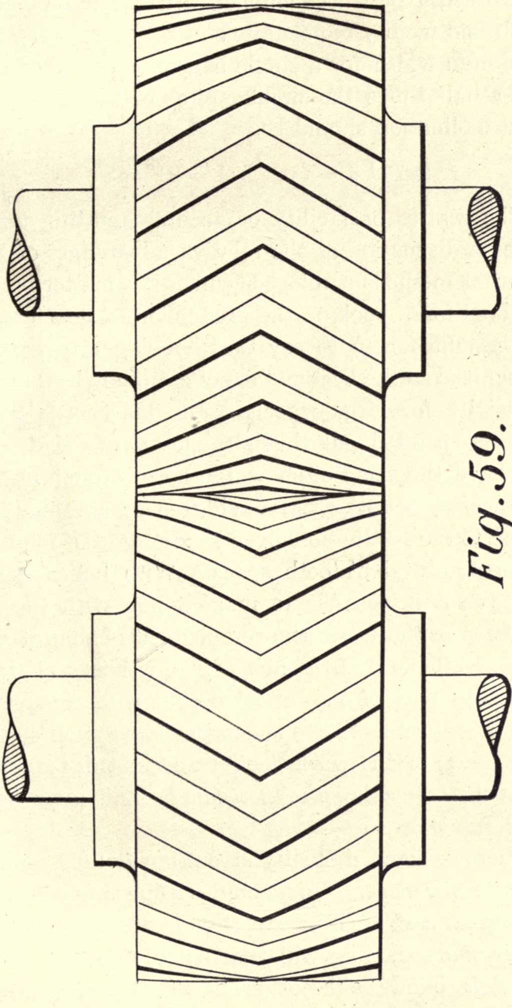


Fig. 59.

it are width of wheel coupled with the pitch of the teeth and facility of accurate cutting. Assuming the minimum width for a single helical wheel to be two and a half times the circular pitch of the teeth, the least inclination should be

$$\tan^{-1}\tfrac{2}{5} = \tan^{-1}\cdot 4 = 21^\circ \cdot 48'.$$

The matter of facility of accurate cutting introduces a discussion of the relative advantages of the different methods employed, which is a matter better reserved for a book on the production of gear teeth; let it suffice here to say that hobbing is generally recognized as the best method of cutting helical teeth, especially for large wheels. In this connexion it may be pointed out that straight spurs cut with a hob have the same pitch as the normal pitch of the hob worm, not its axial pitch (see paragraph 136), owing to the inclination given to the hob axis in order to cut the straight tooth space. With helical teeth however it is possible to make a hob with the inclination of the worm the complement of that of the wheel teeth, so that the hob axis may be set at right angles to that of the wheel blank, with the result that the wheel teeth are cut to the same pitch as the axial pitch of the worm teeth; and in this arrangement the hob axis permits a simpler and more rigid machine.

There is some difficulty in cutting double helical teeth in one piece. On machines using single cutters very great care must be exercised in order to produce a pair of wheels that will bear equally on both right and left hand helix of each wheel; also when machining near the apex, whether with a formed

disc cutter or with a hob, the cutting tool for one side fouls the other set of teeth with the result that the teeth in the neighbourhood of the apex are useless; they are therefore either turned right off just there or the blank is reduced in diameter for a fair width in that part.

In order to reduce this clear space between the helices, Wuest gears "stagger" the teeth, that is the ends of the teeth near the apex on the one side helix are opposite the spaces on the other; this enables the cutter to travel farther before fouling.

As the angle gets steeper the side thrust increases and the normal section decreases with consequent more wear, reduced efficiency and weaker teeth, and no compensating advantage other than perhaps a little smoother running. Narrow wheels demand a steeper angle if all phases of action are to be in operation at any moment.

83. Small Pinions.—To ensure equal contact on both helices one or both wheels are made in two pieces, and the pieces adjusted to proper bearing and bolted together very rigidly. The two-piece arrangement has its disadvantages when applied to a pinion for it limits its smallness, sufficient diameter being required to accommodate the bolts. Where the pinion is in one piece its diameter can be very small, the teeth being cut from the same piece of material as the shaft; the pitch line diameter may then be very little more than the shaft diameter.

Owing to the wonderful smoothness of running of well-cut double helical gears very small pinions working with large wheels can be used, thus giving large velocity ratios and having high pitch line

velocities; pinions as low as five teeth and velocity ratios of twenty to one have been successful and very efficient. When, however, the smallness of the pinion is such that the teeth would be undercut, the pitch is reduced and the wearing surface made good by increasing the width; see paragraph 161 for rules.

Short or "stub" involute teeth with a pressure angle of 20° are used and for pinions below 20 teeth the addendum is increased on the pinion and decreased on the wheel.

To obtain the finest results one of the pair of wheels, the smaller one for convenience, should be permitted a little automatic axial movement, and also a rocking movement about a fulcrum in the central plane of the wheel; the slight inaccuracies that may have crept in during the machining will thereby be compensated, and each side will receive half the load. Gears thus treated and thoroughly well lubricated by a spray on the ingathering side show efficiencies approximating 99 per cent.

CHAPTER X

BEVEL WHEELS

84. Bevel Wheels.—Thus far the wheels treated have been those used to transmit motion between two parallel shafts, and all sections of wheels made by a plane at right angles to the shaft or axis of rotation have been not only similar but equal in every important respect, so that it has been possible to investigate by a plane section only.

It often happens that shafts which are not parallel are to be connected by toothed wheels, and if the axes meet within a reasonable finite distance the problem of the teeth surfaces is not very difficult, though more complex than the simple case of parallel shafts. The wheels used to accomplish this object are known as *bevel wheels.*

In the case where two shafts are neither parallel nor with axes intersecting a single plane cannot be found to contain both, in other words they do not lie in the same plane, the shafts are said to be askew, and the wheels used to connect them are called *skew bevels.* This is a much more difficult problem and the use of such wheels is avoided.

With parallel shafts the important surfaces are right cylinders with axes parallel to the axes of the shafts and have bases formed of the curves already discussed; with bevel wheels all these cylinders

become cones, and the apices of all the cones are at the intersection of the shafts axes. Right circular cones in contact along a slant side and with their apices coinciding will roll upon each other, touching along a straight line, with the same precision as cir-

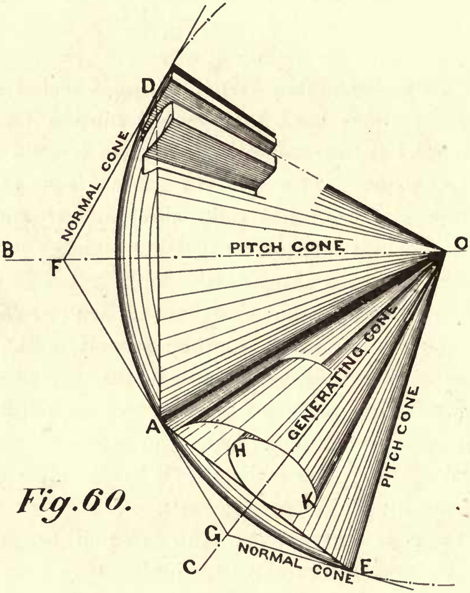


Fig. 60.

cular cylinders, and during the rotation there is no tendency for the apices to separate.

85. Rolling Cones.—In Fig. 60 the two larger cones have contact along OA, their axes are OB and OC, and if fixed upon shafts in these positions one would drive the other, if the frictional grip were sufficient, with angular velocity ratio in inverse proportion to the diameters of the bases AD and AE.

These two cones are called the pitch cones and all important dimensions are made with respect to them. On the upper cone in the picture is shown some teeth which are formed by cutting into and building upon the pitch surface the same as in the cylindrical case. The teeth are of a tapering form, diminishing uniformly towards the apex of the cone, where they become a point.

86. The Tooth Surfaces.—The surfaces of the teeth may be made cycloidal and be swept out by a right circular cone rolling on the inside and the outside of the pitch cones. Such a generating cone is shown in the figure, also a portion of the epicycloidal surface that would be generated by a straight line on the cone as it rolls.

It is not difficult to perceive that all matters of contact, tangency, normals, and arc of action between conical epicycloidal and hypocycloidal surfaces produced by such generating cones hold equally well as between the cylindrical ones for straight spur wheels, the only difference being one of size not of existence; in the conical case all things converge to the apex where they disappear.

It is not at all necessary to use the parts of the cones near the apex, in fact it would be very inconvenient, consequently only a portion near the base is utilized. The diameter of the pitch circle at the large end of the wheel, i.e. the diameter of the base of the pitch cone as illustrated, is the nominal diameter of the wheel.

87. The Spherical Base.—The slant sides of all the cones, whether pitch or generating, are the same length, hence if their apices coincide their bases lie

upon the same sphere, centre O and radius OA in Fig. 60; and a cycloidal curve generated by a point on such a circle as HK also lies on that sphere.

The conditions at the small end of the wheel are the same to a reduced scale.

Strictly the two end surfaces of the wheel teeth should be treated as spherical and made so, and the teeth outlines formed of spherical epicycloids and hypocycloids, but such a treatment for ordinary purposes is unnecessarily troublesome in both draughting and manufacture, and need only be resorted to where great accuracy is called for.

88. Tredgold's Approximation.—In ordinary practice Tredgold's approximation is used. In this the spherical state is neglected and a cone AFD or AGE, Fig. 60, tangent to the sphere at the pitch line (base of pitch cone), and called the normal cone, is used for the end surfaces of the teeth, and upon these normal cones the outlines of the teeth are set out.

The normal cones are found by drawing lines AF and DF at right angles to OA and OD on the upper one, and AG and EG at right angles to OA and OE on the lower one, and their axes coincide with those of their respective pitch cones. A section made by any plane containing the common axis OF of the normal and pitch cones cuts them in straight lines at right angles to each other, hence the name normal cone. The normal cone may be said to be a surface which intersects the pitch cone everywhere normally.

Clearly from the above considerations there is nothing to limit the angle between the shafts; it may be

acute or obtuse, the only restriction is that the axes intersect.

89. Developing the Normal Cones and Setting Out the Teeth.—In Fig. 61 Tredgold's approximate method is given in greater detail. The axes OA and

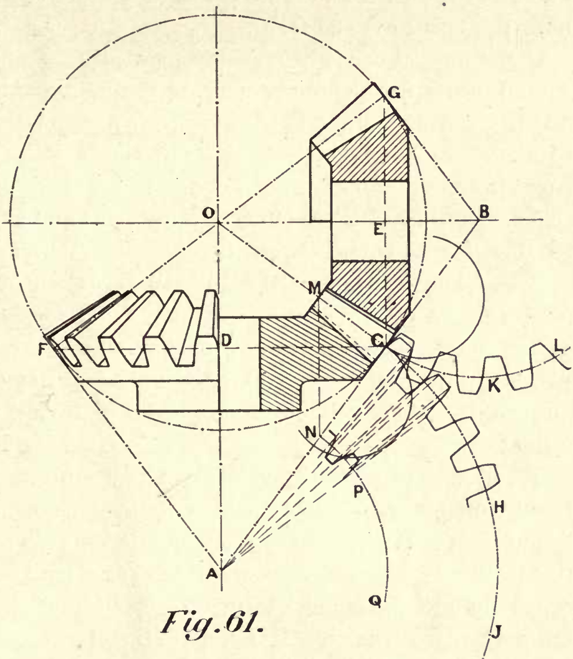


Fig. 61.

OB are at right angles and the wheel on OA is to rotate three times while that on OB rotates four times, the radius DC will then equal $\frac{4}{3}$ CE.

Having decided from considerations of strength and convenience the diameter and size of tooth for say the large wheel, OE may be set out along OB

and OD equal to $\frac{3}{4}$ OE along OA, right angle lines EC and DC drawn meeting at C, and CO joined. CO is the contact line of the two pitch cones, and rotating in turn about the axes it would generate the cones. A line through C at right angles to CO intersects the axes at A and B and determines the normal cones FAC and GBC.

If the normal cones be now developed, that is unwound into a flat sheet commencing at the lines AC and BC, the pitch circle FC develops into an arc CHJ of a circle of radius AC, and extending for a length equal to the circumference of FC, and the pitch circle CG develops into an arc CKL of radius BC and extending for a length equal to the circumference of CG. The two arcs CHJ and CKL drawn as in the figure are tangent at C and may be treated as the pitch lines of two cylindrical wheels, and spaced to provide the required number of teeth and the teeth set out of the cycloidal or involute forms.

90. Replacing the Developed Sheet and the Resulting Error.—If this marking out be done upon thin sheet metal large enough to contain the tops of the teeth and the teeth cut out, the sheets may be coiled up into the conical form and replaced on the normal cones. The arcs CHJ and CKL will coincide with their respective pitch circles and the teeth will gear and touch with almost the same accuracy as on the flat. At the pitch line accuracy will exist, but as the teeth recede from it inaccuracies come in due to unequal distortion of the sheet in wrapping the different parts of the cone.

The shape of the teeth at the small end may be

found by repeating the process for the point M as was performed at C, the whole picture being similar and reduced in the proportion $\frac{OM}{OC}$; or the small end may be found separately as indicated on the arc NPQ by projection from CHJ.

The usual allowance for backlash and clearance is made and the wheels finished off somewhat as in the figure.

To obtain greater accuracy, either the spherical surface should be used or the intersection of the cycloidal surfaces with the normal cones. The latter process is perhaps the better for it admits of the proper curve being obtained on a flat surface, whereas in the former one the sphere baffles all attempts at being flattened and it becomes very difficult to use.

91. To Obtain the True Curve.—To obtain the interpenetration of the true tooth surfaces with the normal cones is laborious. The way that most readily suggests itself is to first find the interpenetration of the generating cone with the normal cone, and then from this interpenetration obtain that of the cycloidal surface in the manner indicated in Figs. 62 and 63.

In Fig. 62 OA is the axis of the pitch cone, OB its slant side; CB is the slant side of the normal cone and is continued indefinitely towards D; OF is the axis and EB the base of the generating cone, and circle centre G is an end view of this base.

The generating cone is continued to penetrate the normal cone, and the interpenetration for the quadrant JH is found and is shown in its developed form at B123. All the construction lines are on the diagram,

but as it is only a lengthy exercise in projection the description in detail is omitted.

Passing to Fig. 63 the curve B123 is transferred from Fig. 62. KBL is part of the developed pitch circle AB ; the circle, centre G, is the base EB of the generat-

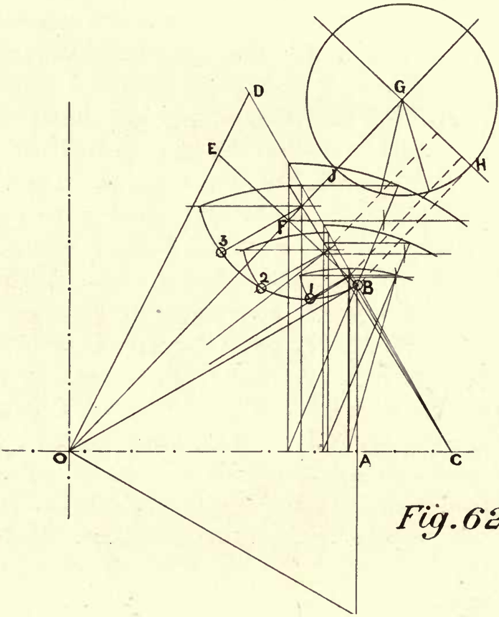


Fig. 62.

ing cone, and although in the position of Fig. 62 its plane is inclined to that of the pitch cone base AB, it rolls upon it and its angular position depends upon the amount of rolling.

The interpenetration curve will move round with the cone but the movement will not be one of rolling but of sliding, the appearance in the development

being as if point B moved round the arc KBL carrying the curve B123 with it.

a, *b* and *c* are the contact points for the trisections of the quadrant of the circle centre G, and *d*, *e* and *f* the corresponding centres. Of the two curves springing up from B the lower one is an epicycloid for a quarter turn corresponding to JH of Fig. 62,

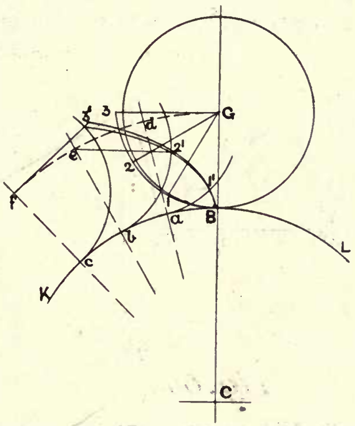


Fig. 63.

and the upper one is curve corrected for interpenetration.

The amount of separation of these two curves indicates the error in Tredgold's approximate method. It will be seen that near the pitch line the error is very small, so that only in the case of large teeth or where great accuracy is sought need it be considered.

It will be noticed that the approximate method produces a curve below the true one for the faces of the teeth, and a similar investigation to that above

would show the flanks a little full, and since the face of a tooth works with the flank of its mate the errors tend to keep the backlash, if any, constant; they do not however keep the normal at the contact point in right direction, none but truly conjugate curves can do this.

Involute teeth set out by the approximate method suffer to the same extent as cycloidal ones, and again by a laborious geometrical exercise in projection the true curve on the normal cone can be obtained.

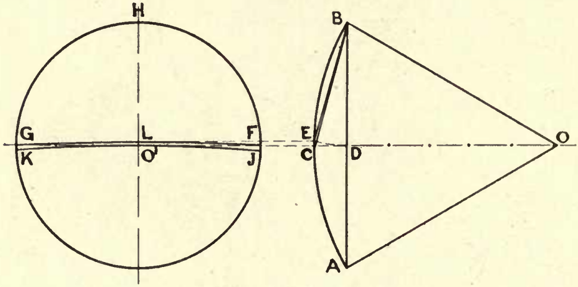


Fig. 64.

92. The Radial Flank.—By Tredgold's method radial flanks may be produced in the development diagram, and these radial lines will still be straight when on the normal cone, so that the flank surface will be on a radial plane of the pitch cone. Now it is impossible to find a spherical hypocycloid to lie in a radial plane.

For considering Fig. 64: If AB be the base of a pitch cone and O the apex, ACB will represent the spherical surface beyond the base. In the circular view GO'F is the trace of a radial plane and the spherical hypocycloid should coincide with it.

If a cone of base diameter BC roll within BOA and the path of C be followed, it will be seen in its central position to be at O′ on GO′F, but when the circle on BC has made a quarter turn either way it will have passed over more than the quadrant HF of the pitch cone base, for BC is greater than BD; the tracing point C will have found a point J or K such that the arc HJ or HK equals the half circumference of the base BC; and the hypocycloid will appear as KO′J.

If a base diameter equal to BD had been chosen for the generating cone, the spherical hypocycloid would be on the radial plane at G and F, but in the middle position would be at L; BE = BD and L is projected from E. From these two cases it is clear that a radial flank cannot be produced by rolling one cone within another.

A spherical hypocycloid might be found to lie on a plane but that plane would not be a radial one of the pitch cone. A radial flank might be made to work accurately if the proper conjugate tooth were obtained by the moulding process described in connexion with straight spur wheels and modified to suit bevel wheels.

93. Internal Bevels.—Internal bevels may be designed by the same methods as external, but it must be borne in mind that secondary action does not occur.

Referring to Figs. 65 and 24 and paragraphs 32 and 33: In 24 it was seen that circles centres A and D produced the same hypocycloid, and that the sum of their diameters equalled the diameter of the director circle centre C; and by the common property of all circles their circumferences have the same ratio as their diameters. In paragraphs 32 and 33 this

equality of hypocycloid produced by either of two circles was used in demonstrating the presence of secondary contact. In Fig. 65 BOA is a pitch cone and C divides the base diameter at any chosen point, the circumferences of circles on BC and CA together equal that on BA. If circles on BC and CA be used as the basis of cones rolling within BOA, the spherical hypocycloids produced are HMK and HNK; these are not identical in shape as can be seen from the picture, hence the arguments respect-

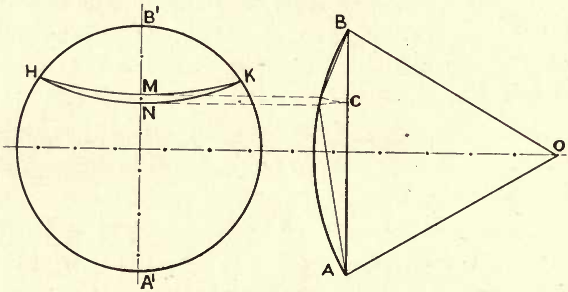


Fig. 65.

ing secondary contact applied to straight spur wheels do not hold for bevel wheels.

Interference will happen at about the same corresponding position as for cylindrical wheels, and it is safe to apply the same rules with respect to it.

94. The Crown Wheel.—When the angle between the shafts is obtuse the velocity ratio required may be such as to make one of the pitch cones so flat that it becomes a disc. In Fig. 66 AB and CD are the axes of the shafts, meeting at 120° at L, the required velocity ratio being 2 to 1. Setting out perpendicular distances 2 from AB and 1 from CD

to any scale and drawing parallels to the axes they meet at E; the line EL is the contact line of the rolling cones. EG at right angles to CD and cutting it in F determines the base of the pitch cone on CD, and from the proportions of the figure EL is at 90° to AB, so that the slant surface of the cone on AB is at right angles to its axis and is therefore quite flat. Twice EL is the outside diameter of the disc.

Applying Tredgold's construction, the normal cone

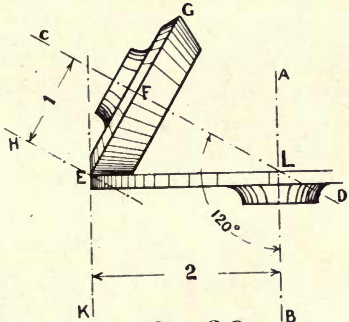


Fig. 66.

to the disc is a cylinder, and in the development the pitch line upon it is straight and the teeth thereon appear as those of a rack.

The type of wheel thus produced is called a *crown wheel*, and is used as the basis of Bilgram's bevel wheel planing machine.

Mitre Wheels.—In the case where shafts are at right angles and the velocity ratio of the wheels is unity, the pitch surfaces of both make an angle of 45° with the axis, the two wheels are alike in every respect, and they are known as "Mitre wheels".

95. Helical Bevel Wheels.—The teeth of bevel wheels may be made helical with the same ease theoretically as ordinary spur wheels, each tooth following a conical helix. Practically there is a little more difficulty especially if pattern moulded; if, however, the teeth are cut from a blank in a bevel wheel shaping machine, all that is necessary is to give the blank the requisite turn as the tool cuts out the space.

CHAPTER XI

SKEW BEVEL WHEELS

96. Skew Bevel Wheels.—The communication of rotation by toothed wheels between shafts whose axes do not meet and are not parallel is overcome by making the pitch surfaces frustra of hyperboloids of revolution. This surface is one generated by a straight line, called the generatrix, rotating about a fixed axis, the distance and the angle between the generatrix and axis remaining constant during the rotation.

97. Hyperboloid of Revolution.—In Fig. 67 C_1D_1 is the plan and CD the elevation of a line capable of rotation about the axis AB (plan O), their common perpendicular OE is the fixed distance apart and the angle BED, in elevation, is the constant angle. A number of positions of the generatrix are shown in both plan and elevation, but the nature of the surface can be better understood from the elevation.

A section made by a diametral plane, as XX, is the two lips of the hyperbola YY and ZZ. By taking a position of the generatrix such that it passes through the plane XX, as the line $m'p'n'$ in plan, and projecting the intersection p' to the elevation at p, one point on the diametral section is obtained, and by repeating for other positions any desired number of points in the curve may be obtained.

The horizontal section through E in elevation is the circle with radius OE_1 in plan, it is at the narrowest part of the surface and is called the gorge circle.

The same surface would have been produced by a line equally inclined in the opposite direction and

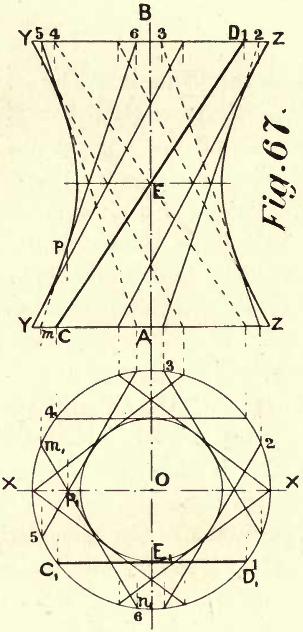


Fig. 67.

rotating at the same fixed distance OE_1 from the axis; thus two systems of straight lines, oppositely inclined, can be found upon the surface.

98. Finding the Rolling Hyperboloids.—In the case of two parallel axes the placing of cylinders upon them to give any desired velocity ratio is a simple matter, but if the axes are inclined and meet

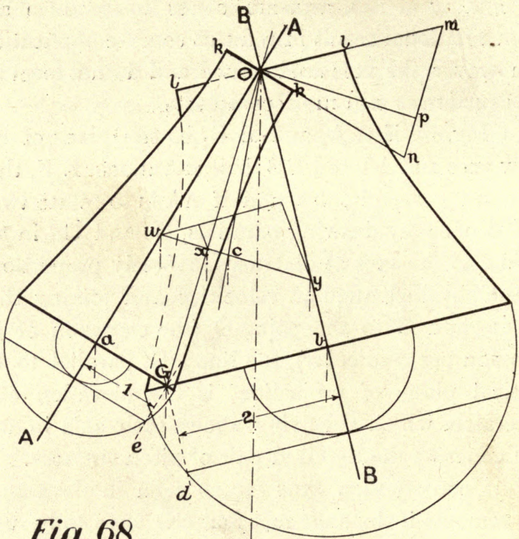
the finding of the requisite cones to produce any given rotational result introduces some complication, if, however, the axes are inclined and do not meet the construction is still more complex.

In Fig. 68 if A_1A_1 and B_1B_1 be the plans of two skew axes and AA and BB their elevations, E_1F_1 their common perpendicular, and if AA is to rotate twice to BB once, by drawing parallels, G*e* and G*d* in elevation, to the axes at distances inversely proportional to the required angular velocities and joining their intersection G to the point O (the elevation of the common perpendicular), the line OG, parallel to the vertical plane of projection, is the elevation of a generatrix which revolving about each axis in turn will generate the required pair of pitch surfaces.

The construction thus far as seen in elevation is the same as if the axes met and GO were to be used to generate cones by revolving about OA and OB as axes, and frustra of the cones used for pitch surfaces of bevel wheels.

It remains now to find the position of OG in plan and thereby completely determine the generatrix.

If in elevation any line as *wxy* be drawn across OG and at right angles to it and terminated by the axes, and E_1F_1 in plan be divided in the same ratio, viz. $\frac{wx}{xy} = \frac{F_1O_1}{E_1O_1}$, the line O_1G_1 parallel to A_1A_1 or B_1B_1 is the plan of the required generatrix. F_1O_1 and E_1O_1 are the radii of the two gorge circles, and the rotation of OG round each axis in turn will generate two hyperboloids which will roll upon each other, with some sliding along the line of contact, and have angular velocity ratio 2 to 1. The surfaces


B
A
m
k
l
O
l
k
p
n
w
x
c
y
a
b
G
1
2
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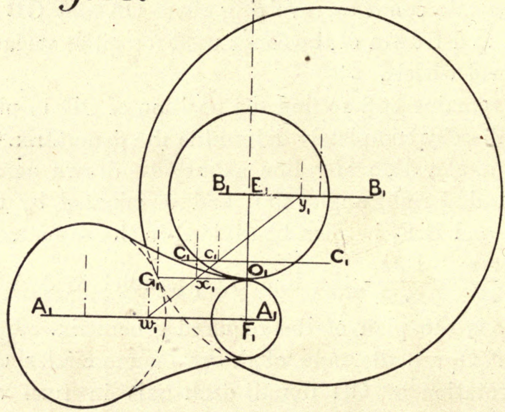
d
B

Fig. 68.


B₁ E₁ B₁
y₁
C₁ C₁ C₁
O₁
G₁ x₁
A₁ A₁
w₁ F₁

are shown in outline in the figure and are limited at their top ends by the gorge circles.

99. Conditions for Rolling and Resulting Velocity Ratio.—The angle between the generatrix and axis AA is GOA and that between the generatrix and BB is GOB,

$$\text{then } \frac{\mathrm{G}a}{\mathrm{GO}} = \sin \mathrm{GOA} \text{ and } \frac{\mathrm{G}b}{\mathrm{GO}} = \sin \mathrm{GOB},$$

$$\text{or} \qquad \frac{\mathrm{G}a}{\mathrm{G}b} = \frac{\sin \mathrm{GOA}}{\sin \mathrm{GOB}};$$

$$\text{and } \frac{xw}{x\mathrm{O}} = \tan \mathrm{GOA} \text{ and } \frac{xy}{x\mathrm{O}} = \tan \mathrm{GOB},$$

$$\text{or} \qquad \frac{xw}{xy} = \frac{\tan \mathrm{GOA}}{\tan \mathrm{GOB}},$$

$$\therefore \qquad \frac{\mathrm{F_1O_1}}{\mathrm{E_1O_1}} = \frac{\tan \mathrm{GOA}}{\tan \mathrm{GOB}}.$$

These equations express the conditions under which the two hyperboloids will roll, and they may be put into words thus: the two hyperboloids will roll together if they coincide along one generatrix and the gorge circles have the same proportion as the tangents of the angles between the generatrix and the axes; the angular velocity ratios will then be in the inverse proportion of the sines of the angles.

100. Angular Velocity Ratio: Proof.—The angular velocity ratio of the two hyperboloids can be shown in the following manner to be as stated above: referring to Fig. 68, *ll* and *kk* are the elevations of the two gorge circles, OG the line of contact of the surfaces, and when one drives the other the linear velocity at right angles to this line must be the same for each.

If Om represent the linear velocity of circle ll, resolving Om at right angles to and parallel to OG, Op at right angles to OG represents the velocity in that direction and mp parallel to OG represents the velocity of sliding along OG; producing Ok and mp to meet at n, pnO is the triangle of velocities for the circle kk, Op being at right angles to OG represents the velocity in that direction, pn that along OG and On that of the circumference of kk, thus

$$\frac{\text{linear velocity of } kk}{\text{linear velocity of } ll} = \frac{\mathrm{O}n}{\mathrm{O}m}.$$

On, Op, Om, and mn are perpendicular to Ow, Ox, Oy, and wy respectively and form a similar set of triangles,

$$\text{then } \frac{\mathrm{O}w}{\mathrm{O}y} = \frac{\mathrm{O}n}{\mathrm{O}m}.$$

Since angular velocity = circumferential or linear velocity ÷ radius

$$\frac{\text{angular velocity of } kk}{\text{angular velocity of } ll} = \frac{\mathrm{O}w \div \mathrm{O}k}{\mathrm{O}y \div \mathrm{O}l} = \frac{\mathrm{O}w \times \mathrm{O}l}{\mathrm{O}k \times \mathrm{O}y}$$

$$= \frac{\mathrm{O}w}{wx} \times \frac{yx}{\mathrm{O}y}, \quad . \quad . \quad . \quad \text{(V)}$$

for the gorge circle radii were made in the proportion $\frac{\mathrm{O}l}{\mathrm{O}k} = \frac{yx}{wx}$; then by similar pairs of triangles Oxw to OaG and Oxy to ObG

$$\frac{\mathrm{O}w}{wx} = \frac{\mathrm{OG}}{\mathrm{G}a} \text{ and } \frac{\mathrm{O}y}{yx} = \frac{\mathrm{OG}}{\mathrm{G}b};$$

substituting these values in V

$$\frac{\text{ang. vel. of } kk}{\text{ang. vel. of } ll} = \frac{\mathrm{O}w}{wx} \times \frac{yx}{\mathrm{O}y} = \frac{\mathrm{OG}}{\mathrm{G}a} \times \frac{\mathrm{G}b}{\mathrm{OG}} = \frac{\mathrm{G}b}{\mathrm{G}a};$$

which was the assumed ratio used in the construction given in paragraph 9.

101. The Teeth of Skew Bevels.—Choosing any point c in the line wxy and joining to O in elevation $\frac{cx}{xO} = \tan COx$; drawing C_1C_1 parallel to A_1A_1 through c_1, the plan of c determines a line which is a third axis about which the generatrix GO may rotate and sweep out a hyperboloid which will roll in contact with either of the first two, inside one and outside the other. In a similar way any number of axes could be found for hyperboloids which would all roll in mutual contact along GO.

Keeping to the axes AA and BB as those of the pitch surfaces, it might seem reasonable that a pair of hyperboloids, with their axes found after the manner of C_1C_1, could be used to roll upon the pitch surfaces and generate cycloidal surfaces that could be used for the faces and flanks of teeth after the fashion of ordinary spur wheels. This theory was accepted as true until Professor MacCord showed in his "Kinematics" that the line of supposed tangency of face and flank was really one of penetration, and in the same treatise he shows how true working surfaces for the teeth may be obtained upon the involute principle. The process is however very complicated, and its explanation beyond the scope of such a book as this. By either method the backs of the teeth are different from the fronts, thus introducing a double operation.

102. Approximate Skew Bevels.—A good serviceable approximation is obtained by assuming the pitch surface the frustrum of the tangent cone at the large

end, and fitting ordinary teeth giving them the direction of the generatrix as in Fig. 69, the upper part of the figure indicates that if the wheel be sufficiently narrow or the frustrum be chosen well away from

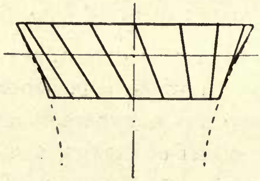


Fig. 69

the gorge the pitch surface is very nearly conical. The operation of cutting the teeth is fairly easily performed in a milling machine, and when cut the wheels may be mounted upon their shafts and run together and the places of interference eased off with a file.

CHAPTER XII

WORM GEARING

103. Worm and Spiral Gearing.—The communication of a rotary motion from one shaft to another where the axes of the shafts are neither parallel nor do meet can be better overcome by the use of worms than by skew bevel wheels, in so far that the working surfaces of the wheel teeth can be made with almost theoretical accuracy with fair ease in the ordinary machines found in an engineer's workshop, and any velocity ratio can be obtained.

In the general case with axes and velocity ratios chosen at random the wheels used would be what are known as *spiral gears* or *screw gears*, the teeth on each wheel being portions of worms or screw threads arranged to mesh one with the other. The diameters of the wheels would not be inversely as the speed of rotation although the number of teeth on each wheel are in that ratio.

104. The Thread or Helix.—In Fig. 70 AA is the axis of a cylinder diameter D; the line 0123 etc., is the elevation of a thread or helix, which is a curve traced upon a cylinder in such a manner that it makes a uniform axial advance as it coils round the cylinder; BC is the axial advance per convolution and is called the pitch of the helix. When treating with screw gears it is convenient to call this the *lead* in

order to avoid confusion with the term pitch applied to the teeth.

If the cylinder be cut along the line BC and unwound into a flat sheet, the base of the cylinder becomes the line $0B' = \pi D$, the points, 0123 etc., develop as shown by the projectors, and the helix itself becomes the straight line $0C_1$. Clearly from the projection the helix is a line of constant inclination a to the base of the cylinder and $90° - a$ to the axis of the cylinder; a is also the inclination to the

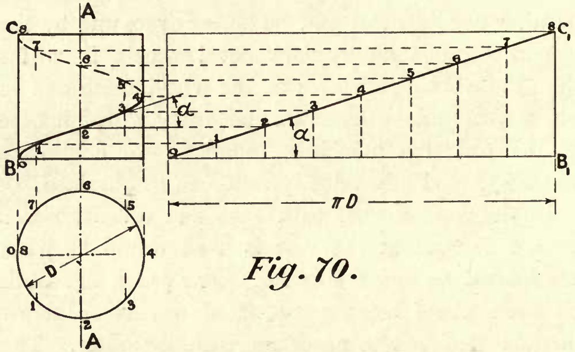


Fig. 70.

plane of the base of the cylinder of a tangent to the helix at any point; the one shown at point 2 in the elevation being parallel to the vertical plane of projection is seen without distortion.

105. The Simple Worm and Worm Wheel.—The simplest case is that in which one wheel has only one tooth and the axis of that wheel at right angles to that of the companion wheel; this arrangement is the common *worm and worm wheel*, and is a very convenient starting point from whence to approach the subject.

In Fig. 71 AA is the axis of a cylinder C upon which are shown three convolutions of a helix, RR is a flat strip which touches the cylinder and is parallel to the axis AA, and on RR is a series of lines having the same pitch as the helix and touching it at *a*, *b* and *c* and laying in the same direction as the helix at the contact points, that is they are set at an angle *α* as shown across the face of the strip.

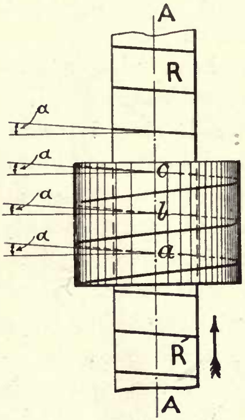


Fig. 71.

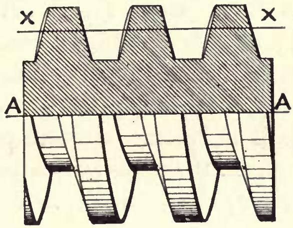


Fig. 72.

If the cylinder carrying the helix be given an anti-clockwise turn viewed in the direction of the arrow, the contact points *a*, *b* and *c* move in the direction *a* to *c* as the different points of the helix come to the bottom, and, if the helix and the lines on RR have the nature of say a fine wire and a groove, the contact point originally at *a* is pushed to *b* during one rotation and on to *c* during the next, and so on, carrying the strip

with it. This is the elementary state of a worm and a rack.

If the strip be bent into a cylinder with axis at right angles to AA and retain contact with cylinder C, the same driving action at the touching point will go on as long as contact is maintained, the circumferential movement of the cylinder RR being equal to the axial movement along C. This is the elementary state of a worm and worm wheel. It only remains to give body to the helix and lines on RR to transform them into a workable mechanical construction.

106. The Worm.—The term helix strictly applies only to a line path, but when substance is given to it it becomes a worm or thread. Fig. 72 represents in the upper part a diametral section and in the lower part an outside view of a worm. If the section be treated as that of a rack, the teeth may have any of the usual forms; the involute is preferable on account of the property of adjustable centre distance and of its straight sides being easy to turn in a lathe.

If XX be the pitch line of the rack section, the revolution of XX about the axis AA will generate a cylindrical surface which will be the pitch cylinder for the worm, and takes the place of the cylinder in the elementary case of Fig. 71. The top and bottom edges of the worm are helices on cylinders of different diameters and will in consequence have different helix angles although having the same axial pitch, but this introduces no complication in considering the action of the worm, all measurement and calculation being made upon the pitch cylinder.

When the worm and worm wheel are properly in gear the pitch cylinder of the wheel touches that of the

worm. The worm can rotate but not move endwise being fitted with a thrust bearing. If a series of diametral sections of the worm be made by the central plane of the wheel as the worm gradually rotates, the sections taken in order present the appearance of a few rack teeth moving along, one complete turn producing a movement equal to one pitch distance of the

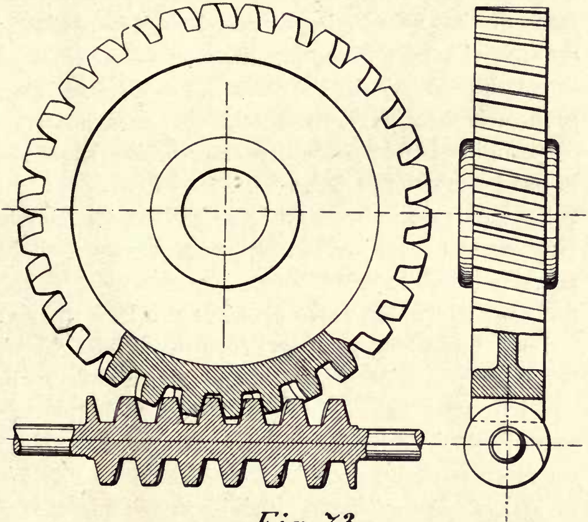


Fig. 73.

rack teeth; the motion may then be very properly looked upon as that of a rack which advances by rotation. The teeth of the rack being in gear with the wheel as the movement takes place, the wheel must rotate.

107. The Wheel Teeth.—The section of the wheel teeth by a plane at right angles to the wheel's axis and containing the axis of the worm should be the same as already described for spur wheels, but

the wheel teeth when fitted must be set obliquely across the face of the wheel at the same angle a as the helix on the pitch cylinder of the worm, see Fig. 73; or more simply the wheel teeth and the worm must have the same direction where they touch.

Contact between the worm and any one tooth will be theoretically at a point only, that point being in the plane just considered where the action is precisely the same as that at a section of a rack and spur wheel; on each side of this section the worm falls clear of the tooth. Practically the contact is over a small area in the neighbourhood of this point, as can be observed on examination of a tooth surface which has been in use for a short period.

Since the area of contact is so limited this simple arrangement is subject to much wear, except for light pressures, and to some extent is responsible for the bad repute in which worm gear has stood for so long.

108. The Close Fitting Worm and Wheel.—A much more serviceable worm and wheel, and one in which contact extends for the whole width of the tooth is obtained by making the wheel teeth lie close up to or partially encircle the worm, as in Fig. 74, instead of merely touching it as in the simple case of Fig. 73. The combination is known as the *close fitting worm and wheel.*

109. Hobbing the Wheel.—If the worm be allowed to cut its own mating teeth in the rim of the wheel, it is natural to expect that the teeth so formed will be perfectly conjugate and the amount of contact would be the maximum possible. This is so, and no better scheme could be possibly devised. It is carried out in practice by making a hob an exact copy, as far

as skill and apparatus will permit, of the worm plus clearance, except for the notching necessary to produce cutting edges.

The hob and a suitable wheel blank are mounted on spindles at the same centre distance as the wheel and worm will occupy when in use, and each is given its proper relative rotation independently. In some of the simpler machines for such work the hob drives the wheel, but it is doubtful whether such an arrangement produces quite as good results as the separate drive. In each case the spaces between the teeth are previously roughed out with a milling cutter, leaving only the finishing cut for the hob.

110. True Form of Wheel Teeth.—Starting with the conception of paragraph 109 it is not a difficult matter to investigate the actual shape of the wheel teeth.

Fig. 74 shows in elevation the usual method of finishing off the sides of the teeth and the rim of the wheel, the angle θ being between 60° and 90°.

If a series of sections be taken parallel to AA and the axis of the worm, the view exposed in each case is that of a rack and wheel gearing together. The central one at AA may be exactly the same as in the simple case of Fig. 73, and is shown at Fig. 75, but no other section will be like it, for none but radial planes can produce in section the straight sides of the worm.

A section taken at BB is shown in Fig. 76; the pitch of the teeth is the same as the axial pitch of the worm, and since the number of teeth on the wheel must be the same as at the central section the pitch diameter of the wheel remains the same. The result

exposed in the section is that of a rack with peculiarly curved teeth profiles, possessing little height and much depth. For accurate gearing, the section of the

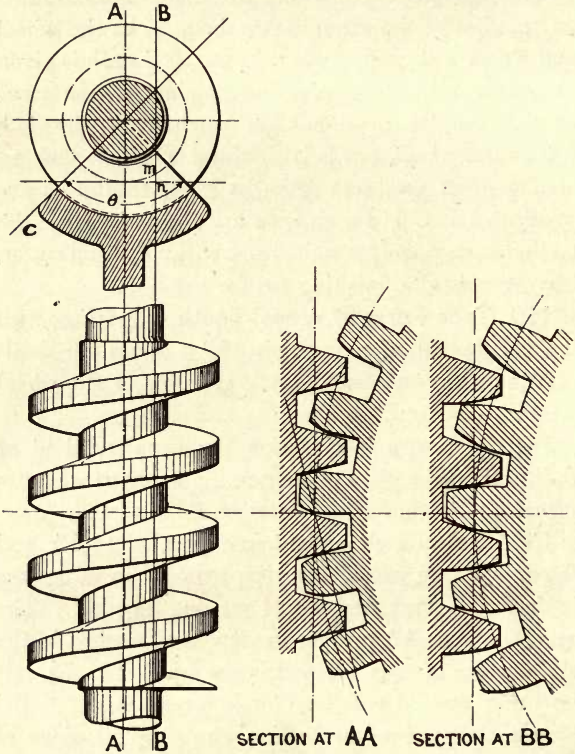


Fig. 74. *Fig. 75.* *Fig. 76.*

wheel teeth at this plane must be conjugate to this peculiar rack, and has been so drawn in the figure.

111. Making the Pattern for Moulding the

Wheel.—If the wheel is to be pattern moulded, the pattern maker must have at least three templates, one for the central section, one for the outside of the wheel, plane CC, and one intermediate as at BB, and the teeth of the pattern cut as near as possible. Templates for one side only of the middle is sufficient as on reversal they will be right for the other side. When the pattern is near completion, the finished worm and the wheel pattern are mounted on spindles and the worm made to turn the wheel, the worm being coated with some suitable marking material; the high places of the wheel teeth are then eased off until a good fit is obtained.

112. Tooth Action Dependent upon Pitch Cylinder.—It should be noted that as the section gets further from the middle the pitch line gets nearer to the point of the rack tooth, for at BB the pitch line of the rack is not through the intersection *m* by the plane BB (see Figs. 74 and 76) with the pitch cylinder of the worm but through *n* and parallel to the worm axis, *n* being on the pitch surface of the wheel which cannot change its diameter since it has a definite number of teeth of constant pitch. When the pitch line reaches the tops of the rack teeth the whole of the tooth action is during approach.

From the above it may be gathered that tooth action is more and more during approach as it gets nearer and nearer to the sides of the wheel.

It has been mentioned earlier that the action during recess is smoother and productive of less frictional loss than during approach, so that if a worm gear of highest efficiency and smoothest working be desired it is well to design with a pitch line at the

central section well down the rack teeth, thereby giving them nearly all face and producing tooth action chiefly during recess. In some worm gears fitted to delicate clock-work the flanks of the worm teeth and faces of the wheel teeth are absent and action is wholly during recess.

With the close fitting worm and wheel, as the contact is along a line instead of at a point, the pressure between the teeth surfaces is correspondingly distributed with the result that the wear is greatly reduced. With hardened steel worms and bronze or even cast-iron wheels and bath lubrication an excellent serviceable gear is produced.

113. Altering Velocity Ratio.—To obtain a quicker drive for the wheel without appreciably altering the size and the number of teeth, and also for reasons of efficiency, worms are made double, treble, and quadruple threaded. There is no need to change the radial section of the worm when multiplying the number of threads, the only element to be varied is the axial pitch or lead. It is here in connexion with multiple threads that the term lead will be used for the axial pitch of the worms and the term pitch applied to the teeth, whether of the rack or the wheel.

114. Multiple Threads.—Fig. 77 serves to illustrate the difference between single and multiple threaded screws. The threads are drawn in the conventional manner with straight lines, which, however, exaggerates the inclination, the true inclinations being shown in Fig. 78; these inclinations being also the obliquities of the teeth across the face of the mating wheel. It is perhaps hardly necessary to point out that one rotation of a double-threaded worm turns

the wheel through two teeth or twice the circular pitch, a treble-threaded worm through three teeth, and so on.

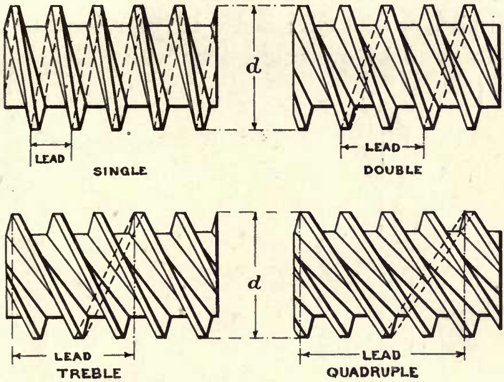


Fig. 77.

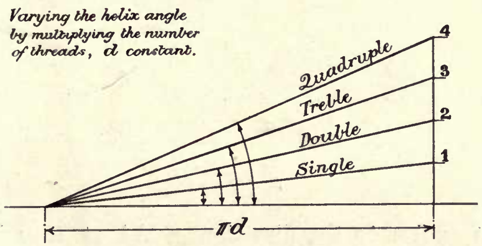


Fig. 78.

115. Approximate Efficiency of Worm.—The manner in which the angle of a worm influences the efficiency of the gear is approximately obtained as follows :—

If in Fig. 79 AA represents a thread making an

angle θ_1 with the line XX at right angles to the axis YY of the pitch cylinder, P the point of contact with a wheel tooth, P to F the direction of axial movement, and PR the development of the thread of axial pitch or lead QR; and if PF represents to scale the force to be overcome, then with PD drawn at

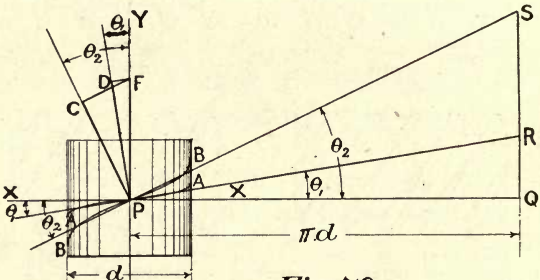


Fig. 79.

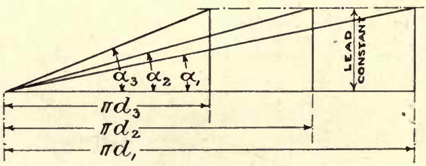


Fig. 80.

right angles to the direction of AA at P and FD parallel to AA at P, PDF is the triangle of forces at P. PD represents the force at right angles to AA or the direct pressure on the tooth and equals PF cos θ_1. PD × coefficient of friction μ is the frictional resistance, and this resistance × the length of one convolution (PR) is the lost work per revolution; PR = πd sec θ_1, therefore

lost work = μ PF cos $\theta_1 \pi d$ sec θ_1 = μ PF πd and useful work = PF × lead QR.

Changing the angle of the thread by multiplying the number of threads, but retaining the same section for purposes of strength, also retaining the same axial force, viz. PF, if BB be the new thread and QS = 3 QR = new lead, then PCF = new triangle of forces, and CP is the pressure at right angles to BB; also CP = PF cos θ_2 and PS the distance travelled by the contact point = πd sec θ_2, therefore

lost work = μ PF cos $\theta_2 \pi d$ sec θ_2 = μ PF πd = same as the previous case, and
useful work = PF × new lead = PF × QS = PF × 3 QR.

In the second case three times the work is done for the same frictional loss as in the first case; the second then is evidently the more efficient.

This investigation, although true as far as it goes, omits certain forces and deals only with the more important ones, but it serves very well to illustrate the influence of helix angle upon the efficiency.

116. Effect of Reduction of Worm Diameter.—Multiplying the threads is not the only means of increasing the helix angle. A reduction of diameter of worm will produce some increase in the angle, see Fig. 80, and since a change of diameter need not alter in the least the radial section of the worm at the thread or teeth, this method is often employed to obtain an increased efficiency. The alteration of the worm diameter will necessitate much change in the form of the teeth in the close fitting wheel, but in the simple tangent one only in the inclination of the tooth across the wheel face.

In designing a worm gear, after having decided the section of the teeth from considerations of the force to be transmitted, the problem is one in which velocity ratio, centre distance, and helix angle are the principal factors.

117. Self-locking Worm.—In some machines using worm drives it is an essential property of the mechanism that it cannot reverse due to any force that may be upon it when stationary, that reversal can be effected only by reversal of the worms rotation by a force supplied from the same source as the forward motion. Such gears are called *self-locking;* and generally a single-threaded worm possesses this property.

Whether or not the worm can reverse due to a stationary load upon it depends upon the coefficient of friction μ and the angle a of the helix. When tan $a = \mu$ it is just self-sustaining at the worm and wheel contact, and taking into account the other frictional resistances of the machine a fair margin of safety is provided.

Referring to Fig. 81, XX is the axis of rotation of a worm and YY its direction at the driving point P, a is the inclination of the worm and ϕ is the angle of friction ($\tan \phi = \mu$), F is the force which applied at P and at right angles to the axis causes rotation.

The forces at P are: the driving force F, the resistance PR of the wheel teeth in their direction of movement, the reaction at right angles to the surface and the friction at the rubbing points, of which the last two combine to form one reaction PB, whose direction is found by drawing first PA at right angles to YY and then PB at an angle ϕ to PA in a

direction opposed to the movement which is towards M when considered in relation to the worm face.

PBR is then the triangle of forces at P, PR representing the force on the wheel teeth in the direction of motion, PB the reaction of the surfaces and BR the applied force F; PR being parallel to the axis and PA at right angles to YY the angle RPA = α the angle of the helix.

Considering the state of possible reversal caused by a force from R to P, if it were to occur, the move-

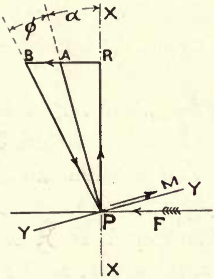


Fig. 81.

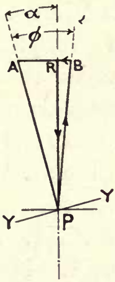


Fig. 82.

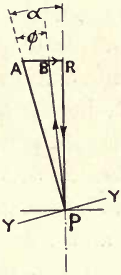


Fig. 83.

ment across the face of the worm would be opposite to M and the reaction PB would be at an angle ϕ on the other side of PA, and again PRB would be the triangle of forces at P. In Fig. 82 it is so drawn and ϕ is made greater than α, with the result that the force at right angles to the axis at P is B to R as the arrows show; and this tends to cause rotation in the same direction as F of Fig. 81, and would be productive of forward rotation which is obviously impossible.

If ϕ be less than α as in Fig. 83 then BR acting

at P tends to push the worm back and reversal may occur.

When $\alpha = \phi$ a neutral condition exists.

118. Advantage of End Thrust on Non-reversal.—With all worm gearing there is a thrust equal to PR along the axis of the worm shaft which must be taken up by a thrust bearing, and another at right angles to it equal to BR which is taken up by the ordinary shaft supporting bearings. Generally the axial thrust is large and the consequent friction at the thrust bearing becomes an important factor and should be included in the calculations when the non-reversal property coupled with high forward driving efficiency is aimed at.

Taking the axial thrust on reverse, whatever be the nature of the bearing that receives it, if the coefficient of friction be known, the moment of friction can be found by the usual methods of textbooks on mechanics, and from the moment thus found the force necessary to overcome it when applied at the pitch circle can be calculated. Calling this new force Q and comparing it with the thrust T as in Fig. 84,

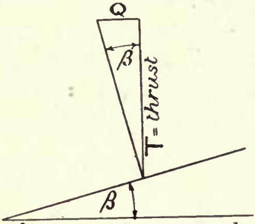
$$\frac{Q}{T} = \tan \beta,$$

the angle of the worm may be increased over and above ϕ by the amount β without destroying the self-locking properties. Altogether then the angle of the thread may safely be $= \phi + \beta$.

All other frictional resistances of the machine might be treated in a similar manner and the angle still further increased if the coefficients of friction for each were known, but as these are variable quantities

depending upon the nature of the surfaces and the state of lubrication, the results would be too uncertain to be incorporated into a design.

119. Efficiency of Worm.—Fig. 81 shows more completely the forces acting at P than does Fig. 79, for the friction is included in 81; referring to that figure, if BR represents in magnitude the force F applied at the pitch circumference of the worm, the work supplied per revolution $= \text{BR}\ \pi d$, where $d =$ diameter of pitch cylinder of worm, PR represents



β = equivalent helix angle

Fig. 84.

the force overcome in direction P to R which in one revolution is for a distance equal to the lead of the worm $= \pi d \tan \alpha$, and useful work done $= \text{PR}\ \pi d \tan \alpha$.

$$\text{Efficiency} = \frac{\text{work got out}}{\text{work put in}} = \frac{\text{PR}\ \pi d \tan \alpha}{\text{BR}\ \pi d}$$

$$\frac{\text{BR}}{\text{PR}} = \tan(\alpha + \phi) \therefore \text{BR} = \text{PR} \tan(\alpha + \phi).$$

Substituting for BR,

$$\text{efficiency} = \frac{\text{PR}\ \pi d \tan \alpha}{\text{PR}\ \pi d \tan(\alpha + \phi)} = \frac{\tan \alpha}{\tan(\alpha + \phi)}.$$

This is the simplest form of the expression for efficiency but involves the angle ϕ instead of the coefficient μ.

By transforming $\tan(\alpha + \phi)$ the equation becomes

$$\text{efficy.} = \frac{\tan\alpha\,(1 - \tan\alpha\tan\phi)}{\tan\alpha + \tan\phi}$$

and putting μ for $\tan\phi$

$$\text{efficy.} = \frac{\tan\alpha\,(1 - \mu\tan\alpha)}{\tan\alpha + \mu} \quad . \quad . \quad \text{(VI)}$$

which is the form generally preferred.

The inclusion of the friction of the thrust bearing in the formula complicates it considerably, but it may be taken that

$$\text{efficy.} = \frac{\tan\alpha\,(1 - \mu\tan\alpha)}{\tan\alpha + 2\mu} \quad . \quad . \quad \text{(VII)}$$

is a fair approximation.

By assigning values to α and μ the efficiency to be expected under any given condition can be worked out. For example assuming $\alpha = 10°$ and $\mu = {\cdot}05$; $\tan\alpha = {\cdot}17633$

$$\text{by equation VI efficy.} = \frac{{\cdot}17633\,(1 - {\cdot}05 \times {\cdot}17633)}{{\cdot}17633 + {\cdot}05}$$

$$= {\cdot}772 \text{ or } 77{\cdot}2 \text{ per cent.}$$

$$\text{by equation VII efficy.} = \frac{{\cdot}17633\,(1 - {\cdot}05 \times {\cdot}17633)}{{\cdot}17633 + 2 \times {\cdot}05}$$

$$= {\cdot}632 \text{ or } 63{\cdot}2 \text{ per cent.}$$

120. Table of Efficiencies.—The results from working a number of cases by equation VI are given in the following tables.

Coefficient of Friction, $\mu=\tan\phi$.	Angle of Worm at Pitch Line $=\alpha$.								
	5°	10°	15°	20°	25°	30°	35°	40°	45°
0·01	89·7	94·5	96·1	97·0	97·4	97 7	97·9	98·0	98·0
0·02	81·3	89·5	92·6	94·1	95·0	95·5	95·9	96·0	96·1
0·03	74·3	85·0	89·2	91·4	92·7	93·4	93·9	94·1	94·2
0·04	68·4	80·9	86·1	88·8	90·4	91·4	92·0	92·2	92·3
0·05	63·4	77·2	83·1	86·3	88·2	89·4	90·1	90·4	90·5
0·06	59·0	73·8	80·4	84·0	86·1	87·5	88·2	88·6	88·7
0·07	55·2	70·7	77·8	81·7	84·1	85·6	86·4	86·9	86·9
0·08	51·9	67·8	75·4	79·6	82·2	83·8	84·7	85·2	85·2
0·09	48·9	65·2	73·1	77·6	80·3	82·0	83·0	83·5	83·5
0·10	46·3	62·7	70·9	75·6	78·5	80·3	81·4	81·9	81·8
0·12	41·7	58·2	66·8	72·0	75·2	77·0	78·2	78·7	78·5
0·14	37·9	54·8	63·4	68·5	71·9	74·0	75·2	75·5	75·0
0·16	34·9	51·0	60·0	65·4	68·9	71·0	72·3	72·6	72·4
0·18	32·2	46·8	57	62·4	66·1	68·3	69·5	69·8	69·4
0·20	29·7	45·2	54·2	59·8	63·5	65·6	66·9	67·1	66·7
	50°	60°	70°	80°	85°				
0·01	98·3	97·8	96·9	94·1	88·5	Zero at 89°·26′.			
0·05	90·2	88·8	84·7	91·0	42·7	Zero at 87°·8′.			
0·20	65·2	58·6	42·0	—	—	Zero at 78°·41′.			

The main table is carried to 45° only as the maximum efficiency occurs at or near that angle, and the efficiencies for angles other than 45° are approximately the same as 90° minus the angle; for example the efficiency at 30° is nearly the same as at 60°, and at 20° as at 70°, the deviation such

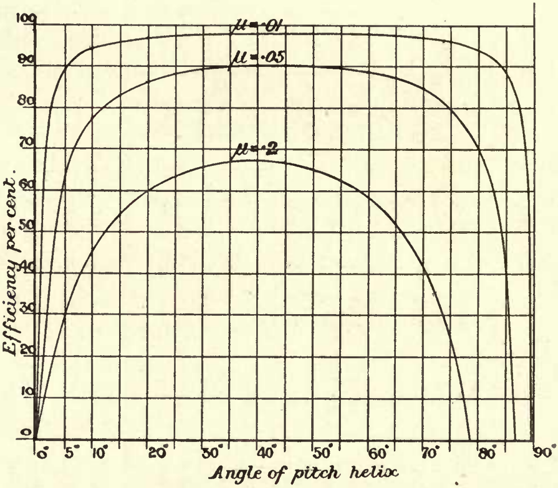


Fig. 85.

as it is being greater the higher the coefficient of friction.

121. Efficiency Curves.—The curves of Fig. 85 exhibit more clearly the changing efficiency with changing helix angle and friction coefficient. Three curves are shown, viz. those for $\mu = \cdot 01$, $\mu = \cdot 05$ and $\mu = \cdot 2$. The first is a condition that may be looked upon as the best attainable with existing material and methods of working, the second may be expected

from a well-finished gear, and the third from a roughly finished and poorly lubricated case.

The most important thing to notice about the curves is the flatness of the top. They show very distinctly that there is a large range of helix angle where the efficiency is not far from the maximum. Taking $\mu = \cdot 05$, which is the most important one, it is clear that helix angles of 20° are little worse than those near 45°, and even at 15° the efficiency still is high, but below this the curve begins to make a rapid descent.

122. Practical Results.—Experiments made by Mr. Wilfred Lewis for Messrs. Wm. Sellers and Co. with worms at various angles, and the results obtained by machine toolmakers adopting worm drives, agree very closely with the theory. All the results point to the fact that there is a best pitch line speed and a limit to the pitch line pressure, but there is insufficient reliable data from which to find the law which governs them. Mr. Lewis' experiments which were conducted upon low angle worms (10° and below) showed definitely that from 200 to 300 feet per minute was the best speed, that 100 feet per minute was also very efficient but below this the falling off was rapid.

Some planer drives designed to run with worm pitch line speeds ranging between 400 and 700 feet per minute cutting stroke and 1000 to 1700 feet per minute return proved great failures. The high velocities were given on account of low angle worms being used, and when these were changed for steeper ones failure became success.

In all worm gears the energy wasted at the rubbing surfaces may be reckoned to be absorbed as heat by the worm and wheel, and if they are unable to convey

it away fast enough to keep the parts cool it is to be expected that heating and rapid deterioration will occur. The matter of lubrication, therefore, is a very important one, for the coefficient of friction depends very largely upon it; metal and metal nearly dry may have a coefficient ·2, and the same surfaces flooded may easily show a coefficient ·05 or less. To get good results then the worm should run in an oil bath.

Some recent results go to show that worm angles in the neighbourhood of 30° will permit of pitch line velocities of 1000 feet per minute with pressures up to 1500 or even 2000 lb., and give excellent results.

A table of figures from Fowler's "Mechanics' Pocket-book" is here given and may be taken to represent the best results yet obtained.

SUMMARY OF VARIOUS RESULTS OF AUTHENTICATED TRIALS (F. BERRY)

Maker.	No.	No. of Threads.	Sliding Velocity, Ft. per Min.	Pressure at Pitch Line in Pounds.	Reduction.	Angle.	Efficiency Per Cent.
Oerlikon	1	3	860	1190	19·5 to 1	24°	93
,,	2	3	1100	412	29·0 ,, 1	23°	87
,,	3	5	765	1179	5·68 ,, 1	31°	94·5
,,	3	5	765	393	5·68 ,, 1	31°	90
,,	3	5	765	1768	5·68 ,, 1	31°	93
,,	4	4	1100	2640	9·7 ,, 1	29°	87
,,	5	3	942	1650	12·0 ,, 1	29° 55′	95
,,	5	3	—	3300	12·0 ,, 1	29° 55′	92
,,	5	3	—	532	12·0 ,, 1	29° 55′	88
D. Brown	6	5	600	1100	10·0 ,, 1	29°	92
& Sons	7	2	1100	7000	20·0 ,, 1	17°	80
German	8	3	540	975	10·0 ,, 1	17° 34′	86
make	8	3	300	—	10·0 ,, 1	17° 34′	88

CHAPTER XIII

OBLIQUE WORM AND WHEEL

123. The Oblique Worm.—When the axes of a worm and wheel are not at right angles the teeth of the wheel are arrived at in much the same manner as when they are; if the worm is not steep a longitudinal section containing the axis AA of the worm is taken and the conjugate teeth obtained, and these set across the wheel face in the direction TT the tangent to the worm pitch helix, see Fig. 86.

124. The Wheel Teeth and Equivalent Spur Wheel.—To obtain the conjugate teeth: a section of the wheel pitch cylinder by a plane containing AA and at right angles to the plane of the paper is made, and the radius of curvature at P of the resulting ellipse found; this radius is then taken as that of a straight spur wheel, called the *equivalent spur wheel*, which gears with a rack whose teeth profiles are the same as the section of the worm by the same plane, and the conjugate teeth for this combination determined.

The radius of curvature at P of the elliptic section is $\frac{r}{\cos^2 \gamma}$, where r is the radius of the worm wheel and γ the complement of the angle between the shafts.

This construction is near enough for pattern-making, but for machine cut wheels, where the cutter travels

in the direction of the teeth, that is parallel to the tangent TT, a section at right angles to that tangent is needed. The section plane in that position makes an angle α with the axis of the worm and the radius of curvature of the section at P is then $\frac{r}{\cos^2(\gamma+\alpha)}$, and this is the radius of the equivalent spur wheel. The construction is shown on Fig. 86.

The line PC at right angles to TT is an edge view of the normal plane to the pitch helix at P, which

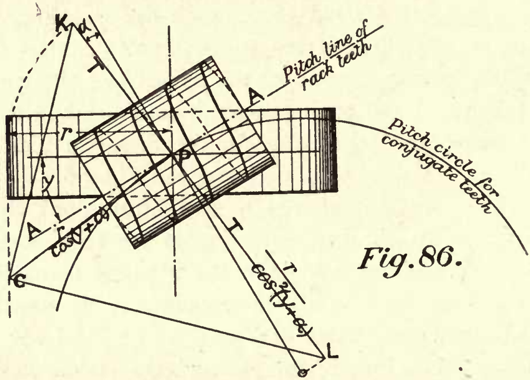


Fig. 86.

cutting the wheel pitch cylinder does so in an ellipse whose major axis is PC and minor axis PK $= r$. Joining CK and drawing CL at right angles to CK meeting TT in L, by similar triangles KPC and CPL $\frac{PL}{PC} = \frac{PC}{PK}$, and $PC = \frac{r}{\cos(\gamma+\alpha)}$ therefore $PL = \frac{r}{\cos^2(\gamma+\alpha)}$.

With a single worm where the helix angle is not large it is sufficiently accurate to treat the normal section as that of a rack, but when the angle gets

large, as it certainly will do with multiple threads unless the worm diameter is proportionately increased, the normal section of the worm near P can no longer be so treated, but the equivalent spur wheel found for it in the same manner as already described for the driven wheel, namely, by finding the radius of curvature at P for the elliptic section made by the normal plane.

Having the two equivalent spur wheel pitch lines, teeth of the form of the normal section at P of the worm are placed upon the worm's equivalent spur wheel at a pitch equal to $p \cos \alpha$, where p is the axial pitch of the worms, and the conjugate teeth obtained for the wheel. (For the several pitches of a multiple worm see paragraph 136.) This is the treatment that screw wheels are subjected to in order to arrive at the correct form of teeth, and is not repeated under that heading. In fact the worm and worm wheel is only a specially simple case of screw gearing; when the helix angle is large or the shafts are not at right angles this simplicity disappears—the contact point cannot be assumed to travel at right angles to the wheel axis.

125. Advance of the Wheel.—With an oblique worm the advance of the wheel circumference is not the same as the lead of the worm per revolution, as may be seen from Fig. 87. For simplicity the wheel may be supposed so great as to be a rack, and the action of the pitch helices examined, the rack being constrained to travel a path not parallel to the worm axis.

The point A is in contact with a tooth XX, and assuming the worm turns as indicated by the arrow,

as rotation proceeds the contact point travels in direction A to B, and at the end of one complete turn has arrived at B. During the operation the tooth XX has been pushed along parallel to itself until it occupies position YY, and A has actually moved to

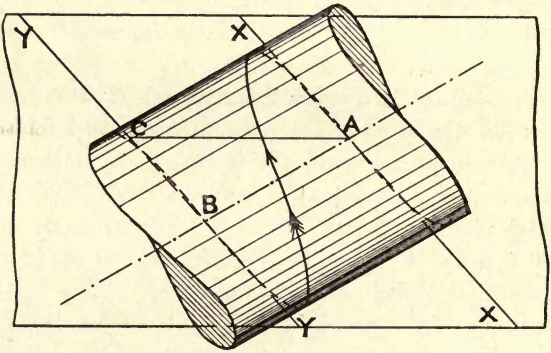


Fig. 87.

C; so that AC is the travel of the rack for one turn of the worm. If instead of a rack a wheel had been used the movement would have been the same amount measured on the pitch circumference.

Fig. 88.

126. Contact of Oblique Worm.—When the teeth are in mesh a vertical section through AB of Fig. 87 would expose a view like Fig. 88 for a worm and rack, for the two are equivalent to two racks, and the conjugate teeth are an exact fit in the spaces. As

the upper rack moves forward its teeth slide through the spaces of the lower one and at the same time push it along, contact point travelling from A to B, so that the section of the driven rack teeth must be the same for the whole of the contact distance C to B.

Putting the wheel in place of the driven rack the teeth must have the same property, namely, of presenting a conjugate face to the worm for the whole of the time of action, and since the worm always presents the same face whether at the nearside, middle or farside of the wheel rim, the wheel must do likewise, so that the section of the wheel teeth must be same right across. Towards the sides of the wheel where the worm cylinder is clear of the wheel cylinder the teeth may be carried a little higher to obtain a longer contact; but the profile must be a continuation only of that at the middle. Just how high they may be carried can be found from the diagram Fig. 86 used to obtain the wheel teeth.

Since the wheel teeth must have the same section right across with the exception of the slight increase in height just mentioned, this type of wheel cannot be made "close fitting". Contact with any tooth can only be at a point so that the line contact of the close-fitting wheel can never be obtained.

127. Effect of the Helix Angle on the "Hand" of the Wheel Teeth.—The effect of changing the helix angle of the worm upon the direction of the wheel teeth is worthy of notice.

With axes at right angles and a given direction of rotation of the worm, a right-handed worm drives the wheel in one direction and a left-handed worm drives

it in the opposite direction. In each case the teeth being set obliquely across the wheel face they also become portions of worms upon the wheels pitch cylinder and of the opposite hand to the driving worm. These conditions hold however much the threads are multiplied or however the diameter of the worm is altered.

With oblique axes it will be seen on reference to

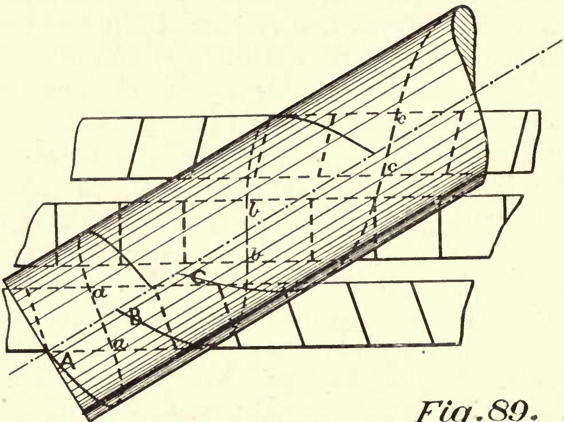


Fig. 89.

Fig. 89 that the hand of the wheel teeth depends upon the angle of the driving worm and obliquity of the axes. In that figure are shown three right-handed threads of different inclinations, and the directions of the teeth to gear with them. *aa* is the direction of the tooth for thread A and is part of a left-handed helix on the wheel, *bb* is the direction of the tooth to suit thread B and is straight across so that a common spur wheel would serve, and *cc* which suits thread C is part of a right-handed helix on the wheel.

In each of the three cases a given direction of rotation of the worm produces the same direction of rotation of the wheel notwithstanding the different hand of the wheel teeth.

128. Effect of Multiplying the Threads on the Pitch of the Wheel Teeth and on Velocity Ratio.— Another difference between right angled and oblique axes is that whereas in multiplying the threads in the right-angled case the velocity ratio is reduced in the inverse proportion if the same wheel diameter be retained, with oblique axes this does not hold true, for the pitch of the teeth will change as the number of threads is increased.

In Fig. 90 P*abc*A shows a helix which may be looked upon as a worm driving the wheel set obliquely below it. In one rotation the driving point P will have travelled to A. Setting off PO = πD at right angles to the worm axis and joining PA, POA is the inclination of the helix and OA is the direction of the wheel teeth, P to N is their direction of movement and PN is their distance travelled corresponding to the movement of P to A.

Doubling the number of threads without altering the radial section will double the lead of the helix, and such a change is seen at P*def*B where PB = 2 PA. Applying the same construction as to the original helix, PB is the travel of the driving point on the worm per rotation and PM that of the driven point on the wheel. Inspection of the diagram will show that PM is less than twice PN; geometrically, for PM to equal 2PN then AN and BM must be parallel, and to obtain this the diameter of the worm cylinder must be doubled when the thread is doubled.

The result then is that doubling the pitch of the helix does not double the circumferential movement of the wheel. To obtain the helix that will make PM twice PN, set out PM equal to twice PN and join O to M and produce to intersect the axis of the

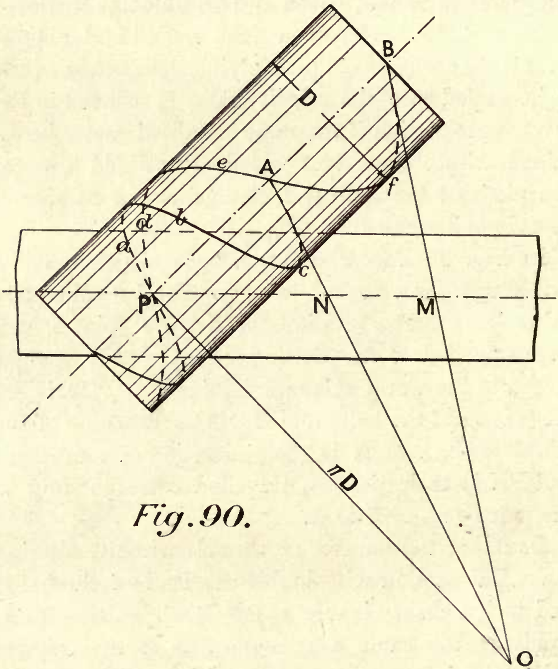


Fig. 90.

worm, then from P to the intersection is the required lead.

Moreover in the first case PN is the circumferential pitch of the wheel teeth, and in the second case PM is twice the pitch since the worm is doubled; the diameter of the wheel must therefore be reduced

in the proportion $\frac{PM}{2PN}$ if the velocity ratio is to be half the original. If the object of the change were only to obtain increased efficiency by the steeper worm angle, retaining the original velocity ratio, then the wheel diameter would have to be increased in the proportion $\frac{PM}{PN}$.

This non-regular condition of pitch, wheel diameter and velocity ratio is discussed more fully under screw wheels, to which the worm with oblique axis really belongs; it has been introduced here only as a stepping-stone from the simpler worm and worm wheel.

129. The Terms "Pitch" and "Lead".—With single-threaded worms the term pitch applies to the axial advance per rotation, but with multiple threads this term has not in common practice quite the same meaning. The term *lead* is used to indicate the axial advance per rotation and the term pitch retained for the distance between the crests of two consecutive threads. In Fig. 77 the pitch used in this sense is the same for all the four worms, and is the same as the lead of the single-threaded one, while the lead in each is in the proportions 1 : 2 : 3 : 4. This arrangement of names is convenient in that "pitch" has reference to tooth forms in every case and applies equally well to single or multiple threaded worms or to wheels, and "lead" refers to the axial advance of a helix for one convolution.

CHAPTER XIV

SCREW WHEELS OR SPIRAL WHEELS

130. Screw Wheels or Spiral Wheels.—If the velocity ratio is fairly high and it is never intended to drive the worm by way of the wheel, the gear is known as a worm and wheel, but if the velocity ratio is nearer unity and the two rotating pieces approximate the same form, and each is equally capable of being the driver, the wheels are then known as *screw* or *spiral wheels*.

The velocity ratio is always inversely as the number of teeth, but the circular pitch of the teeth can be varied, not being the same on the two wheels except under conditions which will appear later, consequently wheel diameter is no guide to velocity ratio.

131. Similar Properties to the Oblique Worm and Wheel.—In paragraph 128 and Fig. 90, it was pointed out that doubling the lead of the worm reduced the pitch of the wheel teeth; trebling the lead and the number of threads would give a still further reduction and so on. Evidently the pitch of the wheel teeth is bound up with the angle and lead of the worm. If the worm had been of greater diameter and of the same radial section, the angle would have been less, but the influence of multiplying the threads and thereby the lead of the worm would still have

affected the pitch of the wheel teeth, though not to the same extent. These same properties hold in screw wheels.

132. Some Advantages of Screw Wheels.—Some advantages of screw gearing are: its silent and smooth running: that subject to limitations the velocity ratio is independent of the diameter: and that non-parallel non-meeting shafts can be connected by wheels whose teeth are easily constructed with accuracy.

133. Growth from a Simple Worm and Worm Wheel.—In the first paragraph of this book, dealing with kinematic transformations, it was pointed out and illustrated that the simple worm and wheel might grow into a pair of screw wheels; repeating now in detail with the aid of some definite dimensions the growth may be more apparent.

Starting with a single worm of 10″ diameter gearing with a wheel of 10″ diameter and 20 teeth, the velocity ratio is 20 to 1. Doubling the threads of the worm and retaining the same radial section will give a ratio 10 to 1, quadrupling will give 5 to 1 and so on; with 10 threads the ratio is 2 to 1 and the angle of the wheel teeth is such that there is no great difference in the appearance between them and the threads of the worm. Cutting away such portions of the worm as are unnecessary, there exists a pair of wheels much alike with equal diameter but giving a velocity ratio 2 to 1. Still further increasing the threads to 20, the ratio is 1 to 1 and there is now no difference between them, the teeth lay across the faces of the wheels at 45°, the original wheel may change places with the original worm and the transformations

carried in the reverse order until the wheel becomes the single worm and the worm a wheel of 20 teeth.

If instead of the worm being 10″ diameter it had been 5″ and the wheel still 10″ diameter and 20 teeth, the angle of the thread would have been 45° when it had been multiplied ten times, and 63° 24′ when multiplied twenty times, and this latter angle is sufficiently steep for efficient driving by either worm or wheel, so that two wheels may exist of diameters 2 to 1 and velocity ratio 1 to 1.

With the diameters 3 to 1 the worm angle for velocity ratio 1 to 1 is 70° 33′, which is approaching the limit for driving by either wheel.

The above examples demonstrate that theoretically any diameters for worm and wheel might be chosen, and by transformations any velocity ratio obtained, and that practically there is a fair range of choice.

134. Velocity Ratio, Axes at Right Angles.—If a pair of screw wheels for axes at right angles be made and the cylinders of teeth be capable of being unwound each into a flat sheet, and they be so treated and the teeth laid in gear, the appearance of the development will be as in Fig. 91, where XX is the axis of the wheel A and YY that of B, NN the common normal to the helices at the pitch point P, θ_1 the angle between NN and YY which equals the angle between the teeth of A and its axis XX, and θ_2 the angle between NN and XX which equals the angle between the teeth of B and its axis YY. θ_1 and θ_2 are commonly called the screw angles or spiral angles.

Treating as two racks for simplicity: when driving occurs the linear velocity for both must be the same

in the direction NN or otherwise penetration would occur or one of the pieces would deviate from its straight path at right angles to its axis. If the driving point on A moves from P to S the simultaneous movement in direction NN = PR, and PS $= \dfrac{PR}{\cos \theta_1}$. For the same normal movement on B the driven point on B must have moved from P to Q, and

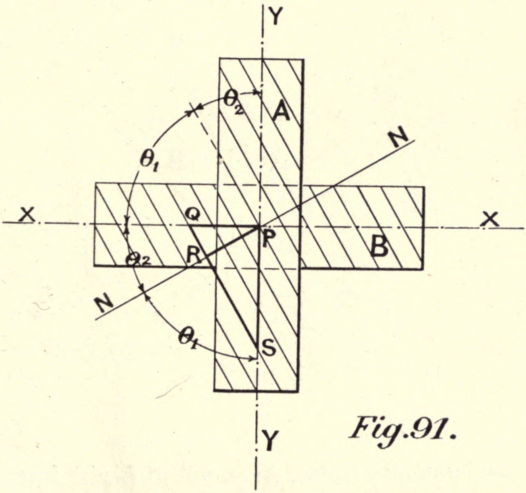


Fig. 91.

PQ $= \dfrac{PR}{\cos \theta_2}$. A movement of PS on A then is accompanied by one of PQ on B, and $\dfrac{PQ}{PS} = \dfrac{\cos \theta_1}{\cos \theta_2}$.

If radius of A $= r_1$, radius of B $= r_2$

$$\frac{\text{Ang. vel. of A}}{\text{Ang. vel. of B}} = \frac{PS}{r_1} \div \frac{PQ}{r_2} = \frac{PR}{\cos \theta_1 r_1} \div \frac{PR}{\cos \theta_2 r_2}$$

$$= \frac{\cos \theta_2 r_2}{\cos \theta_1 r_1}. \qquad \text{(VIII)}$$

or in words: *the velocity ratio varies inversely as the products of the radius of each wheel and the cosine of its screw angle.*

It must be noted that here the screw angle is the complement of the inclination of the thread as treated on worm gears.

With axes at right angles as in Fig. 91 θ_1 and θ_2 are complementary, that is $\theta_2 = 90^\circ - \theta_1$.

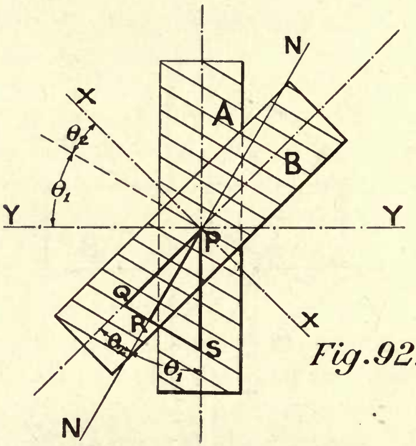


Fig. 92.

135. Velocity Ratio, Axes not at Right Angles.—When the axes are oblique the expression for velocity ratio still holds, and the explanation of Fig. 91 can be read in connexion with Fig. 92; $\theta_1 + \theta_2$ however no longer equals 90°, but equals XPY the angle between the shafts.

136. Relation Between the Pitches.—In any screw wheel there are three different pitches to consider as may be seen in Fig. 93. Assume the helices there shown to be those of the teeth of a screw wheel,

the useful portion being that bounded by the lines DD and EE. Measuring circumferentially from thread to thread the distance PC is known as the *circumferential pitch*, measuring at right angles to the thread PN is called the *normal pitch*, the distance HK is the axial pitch of the helix and better called the *lead*, while PA measured in an axial direction is the *axial pitch of the teeth.*

$$\text{PC} \times \text{number of teeth or helices } (n) = \text{circumference of pitch cylinder}$$

$$\frac{\text{PC} \times n}{\pi} = \text{diameter of pitch cylinder } (d), \text{ or PC} = \frac{\pi d}{n},$$

$\text{PN} = \text{PC} \cos \theta$, where θ = angle between the helix and the axis of the cylinder, or the screw angle,

$\text{HK} = \pi d \tan (90° - \theta) = \pi d \cot \theta$ (see paragraph 104 and Fig. 70), and

$$\text{PA} = \frac{\text{HK}}{n} = \frac{\pi d}{n} \cot \theta; \text{ therefore}$$

$$\text{PC} : \text{PN} : \text{PA} :: 1 : \cos \theta : \cot \theta.$$

Fig. 94 is a development of a part of the cylinder and helices of Fig. 93 and shows the distances PC, PN, and PA without distortion, and the trigonometrical relations can be read directly from the diagram.

137. Form of Teeth.—It is hardly necessary to demonstrate the fact that when one set of teeth gears with another set, whether straight across the face as in common spurs or obliquely as in screw wheels or oblique racks, that the shortest way through a tooth and across a space, reckoned on the pitch surface, is at right angles to the faces, and that the shortest way through a tooth exactly fits the shortest way across the space, backlash neglected, and more important

still the sum of these shortest distances is the same for both the gearing wheels, or wheel and rack or two racks.

This sum of least thickness and space being at right

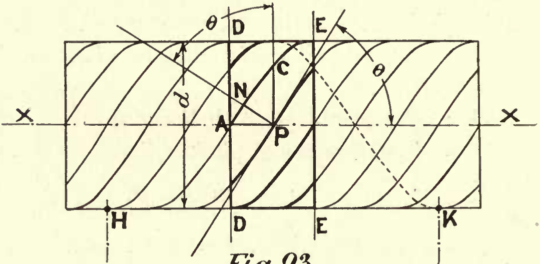


Fig. 93

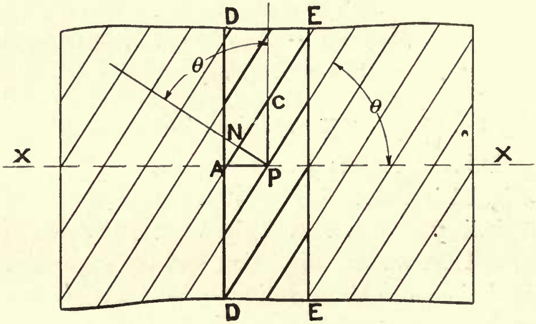


Fig. 94.

angles to the faces is called the normal pitch ; and the point to be given prominence is that the normal pitch of a pair of screw wheels is the same in each whatever the circular pitches may be.

In the design of screw wheel teeth the section along the normal plane at the pitch point is considered in

the manner described for the oblique worm and wheel (paragraph 124), but with screw wheels it is not usual to treat one as a rack with involute teeth but each as a spur wheel, and in the normal section the teeth are given the ordinary profile of spur wheel teeth, cycloidal or involute, suited to the normal pitch and the radius of curvature of the elliptic section at the pitch point.

138. Selecting the Cutter.—Where standard spur wheel milling cutters are employed in the manufacture of screw wheels the problem of selection is settled by choosing that one which will give the nearest correct profile for a spur wheel tooth having the same pitch as the normal pitch of the screw wheel and a number of teeth equal to $\frac{n}{\cos^3 \theta}$, where n is the number of teeth in the screw wheel and θ is the screw angle.

This is arrived at thus: circumferential pitch of screw wheel = normal pitch $\div \cos \theta$, then circumferentially the normal pitch would step round $\frac{n}{\cos \theta}$ times: radius of the elliptic cut made by the normal plane at the pitch point $= \frac{r}{\cos^2 \theta}$, where r is the radius of the screw pitch cylinder: the equivalent spur wheel of this radius would then contain

$$\frac{n}{\cos \theta} \times \frac{1}{\cos^2 \theta} = \frac{n}{\cos^3 \theta}$$

teeth of the screw's normal pitch.

139. Interdependence of Dimensions.—The normal section of a tooth having been decided from considerations of strength to resist the force transmitted, and from the nature of the material, the

normal pitch is fixed within narrow limits. Usually also the centre distance cannot be much altered and the shaft angle and velocity ratio are fixed quantities. Three other dimensions, namely wheel diameter, screw angle, and number of teeth are not of such a rigid nature and may generally be anything that will suit the design. All the seven are, however, so intimately connected that a change in one may necessitate a change in several if not all of the others.

It is not possible by a straight forward process of algebra or geometry to solve any given problem in screw wheels. Two processes are necessary; the first solves approximately, and the second by trial and error finds a result within the possible accuracy of manufacture.

140. Equations Connecting Dimensions.—The following equations connecting the various important dimensions follow and are useful in the algebraic solutions. Reference to Figs. 91 to 94 and paragraphs 134 to 136 will be useful in identifying the symbols.

$$\text{Velocity ratio} = \frac{\text{revs. of A}}{\text{revs. of B}} = \frac{\cos\theta_2 r_2}{\cos\theta_1 r_1},$$

c_1 = circum. pitch of A = normal pitch (p) $\div \cos\theta_1$,

$$\text{or } c_1 = \frac{p}{\cos\theta_1},$$

c_2 = circum. pitch of B = normal pitch (p) $\div \cos\theta_2$,

$$\text{or } c_2 = \frac{p}{\cos\theta_2},$$

$$\left.\begin{aligned} d_1 &= 2r_1 = \text{diam. of A} = \frac{n_1 c_1}{\pi} \\ d_2 &= 2r_2 = \quad \text{,,} \quad \text{B} = \frac{n_2 c_2}{\pi} \end{aligned}\right\}$$ where n_1 and n_2 are the numbers of teeth on A and B respectively,

$$\text{then } r_1 = \frac{n_1 c_1}{2\pi} \text{ and } r_2 = \frac{n_2 c_2}{2\pi};$$

or by substitution for c_1 and c_2

$$r_1 = \frac{n_1 p}{2\pi \cos \theta_1} \text{ and } r_2 = \frac{n_2 p}{2\pi \cos \theta_2},$$

$$\text{then } \frac{r_1}{r_2} = \frac{n_1 \cos \theta_2}{n_2 \cos \theta_1}.$$

$\theta_1 + \theta_2 =$ shaft angle, and when shaft angle $= 90°$, $\cos \theta_2 = \sin \theta_1$ and $\cos \theta_1 = \sin \theta_2$.

141. Worked Examples (Numerical).—EXAMPLE I. Given shaft angle 90°, velocity ratio to be 2 : 1, spiral angle 45°, normal pitch ·5″ and 20 teeth on B; find diameters and centre distance.

$$\text{Cos } \theta_1 = \cos \theta_2 = \cos 45° = 1.$$

Velocity ratio $= \dfrac{\cos 45° \, r_2}{\cos 45° \, r_1} = \dfrac{r_2}{r_1} = \dfrac{2}{1}$, which is in this case inversely as the diameters or radii.

$$\text{Diam. B} = 2r_2 = \frac{2 \, n_2 p}{2\pi \cos \theta_2} = \frac{20 \times \cdot 5 \times \sqrt{2}}{3 \cdot 14} = 4 \cdot 5''.$$

Diam. A $= 2r_1 = \frac{1}{2} \times 4{\cdot}5 = 2{\cdot}25''$.

Sum of diams. $= 6{\cdot}75$; and centre distance $= 3{\cdot}375''$.

If this dimension is suitable all is well, but if the condition had been given that centre distance was to be 3·5″ some adjustment must be made. Trying a change of angle, let $\theta_1 = 40°$ and $\theta_2 = 50°$, and substituting accordingly.

$$\text{Diam. B} = \frac{2 \, n_2 \, p}{2\pi \cos 50°} = \frac{20 \times \cdot 5}{3 \cdot 14 \times \cdot 643} = 4 \cdot 96''.$$

$$\text{Diam. A} = \frac{2 \, n_1 \, p}{2\pi \cos 40°} = \frac{10 \times \cdot 5}{3 \cdot 14 \times \cdot 766} = 2 \cdot 077''.$$

Sum of diams. $= 7{\cdot}037$; and centre distance

$= 3{\cdot}513''$,

which is ·013″ too much and should not be allowed to pass.

Trying $\theta_1 = 40° \, 15'$ and $\theta_2 = 49° \, 45'$.

$$\text{Diam. B} = \frac{2\,n_2\,p}{2\pi \cos 40° \, 15'} = \frac{20 \times \cdot 5}{3\cdot 14 \times \cdot 649}$$
$$= 4\cdot 905''.$$

$$\text{Diam. A} = \frac{2\,n_1\,p}{2\pi \cos 49° \, 45'} = \frac{10 \times \cdot 5}{3\cdot 14 \times \cdot 763}$$
$$= 2\cdot 088''.$$

Sum of diams. $= 6\cdot 993$; and centre distance
$$= 3\cdot 496'',$$

which is $\frac{4''}{1000}$ short, and is near enough for practical purposes.

If an adjustment of the normal pitch had been permissible the correction of centre distance would have been easier. The normal pitch would have been reduced in the ratio 3·75 to 3·5, and the new diameters found by simple proportion. Usually, however, the normal pitch is not amenable to *slight* adjustments, for in most cases teeth have to be cut with standard cutters, and the manufacturer does not possess an unlimited range of these. If a special cutter be made for any particular case of course the difficulty no longer exists.

If the shaft angle be other than 90° and the direction of the screw teeth bisects the angle between the shafts so that $\theta_1 = \theta_2$ then velocity ratio $= \frac{r_2}{r_1}$, or inversely as the diameters. In this condition only, namely equal screw angles, does the simple relation of velocity ratio being inversely as the diameters hold good.

EXAMPLE II. Given shaft angle 90°, velocity ratio 1 : 1, screw angles 30° on A and 60° on B, normal pitch ·5″ and 20 teeth on each A and B ; find suitable diameters and centre distance.

$$\text{Cos } 60^\circ = \cdot 5, \cos 30^\circ = \cdot 866.$$

$$\text{Velocity ratio} = \frac{\cos\theta_2\, r_2}{\cos\theta_1\, r_1} = \frac{\cos 60^\circ\, r_2}{\cos 30^\circ\, r_1} = \frac{\cdot 5\, r_2}{\cdot 866 r_1} = 1,$$

$$\text{therefore } \frac{r_1}{r_2} = \frac{\cdot 5}{\cdot 866}.$$

$$\text{Diam. B} = \frac{2\, n_2\, p}{2\pi \cos 60^\circ} = \frac{20 \times \cdot 5}{3\cdot 14 \times \cdot 5} = 6\cdot 37''.$$

$$\text{Diam. A} = \text{diam. B} \times \frac{r_1}{r_2} = 6\cdot 37 \times \frac{\cdot 5}{\cdot 866}.$$
$$= 3\cdot 676''.$$

Sum of diams. $= 10\cdot 036''$; and centre distance $= 5\cdot 018''$.

If this is unsuitable adjustment can be made as in example I ; or if the discrepancy is large a change in the number of teeth, retaining the proper ratio, may be more suitable as a first step towards correction.

Usually in the case of shafts at 90° and velocity ratio 1 : 1 the screw angles are 45° on each wheel, and the wheels are of equal diameters, but where equal diameters cannot be arranged for some such condition as in example II arises.

These examples serve to show the kind of calculation that has to be made for the determination of all the dimensions, and the dependence of one quantity upon another.

142. Graphical Method of Solving.—The solution of screw wheel problems by the geometrical method are still subject to the trial and error process, and to the inaccuracies of all graphical work, but a

better mental conception of the amount of dependence of one dimension upon the other is obtained; the two wheels are practically before the eyes and the effect produced by a change in any one part can be more readily seen.

Fig. 95 shows the developments of two screw wheels laid in contact at their pitch point P. Pa =

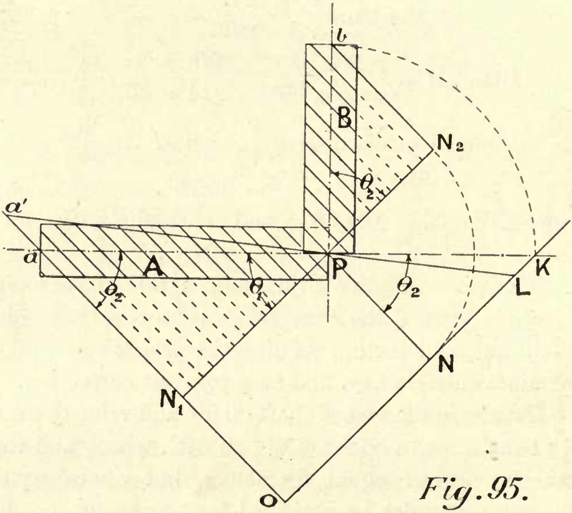


Fig. 95.

circumference of A and Pb that of B, PN_1 is the normal crossing all the threads of A and is the developed normal helix, PN_2 is the same for B, and since the linear velocities along the normal helix are the same for both wheels $\frac{\text{revs. of A}}{\text{revs. of B}} = \frac{PN_2}{PN_1}$,

also aPN_1 equals the screw angle of A $= \theta_1$

and bPN_2 equals the screw angle of B $= \theta_2$,

$$PN_1 = Pa \cos \theta_1 \text{ and } PN_2 = Pb \cos \theta_2,$$

then by substitution

$$\frac{\text{revs. of A}}{\text{revs. of B}} = \text{velocity ratio} = \frac{\text{P}b \cos \theta_2}{\text{P}a \cos \theta_1};$$

since circumferences are in the same proportion as radii $\frac{\text{P}b}{\text{P}a} = \frac{r_2}{r_1}$ then velocity ratio $= \frac{\cos \theta_2 \, r_2}{\cos \theta_1 \, r_1} = \frac{\text{PN}_2}{\text{PN}_1}$, which is the same form as equation VIII.

If now $b\text{PN}_2$ be turned through a right angle in the direction indicated it will occupy position PNK, and continuing KN and $a\text{N}_1$ to meet at O, NON_1P is a rectangle whose sides are in the proportion of the velocity ratio, and any rectangle having its diagonal on OP and sides along ON_1 and ON will be similar. Since PK = Pb, aK is divided at P in the proportion of the circumferences or diameters or radii of the two wheels. If the whole diagram be reduced in the proportion π to 1 circumferences become diameters.

Any adjustment of centre distance can now be made by taking the sharp straight edge of a piece of paper, passing it through P varying its position until the length intercepted between the lines Oa and OK, produced if necessary, equals twice the required centre distance. If a'L be the result of such an operation, the length a'L being twice the centre distance, the point P divides it in the proportion of the diameters, and the direction of a'L gives the new screw angles $a'\text{PN}_1$ and LPN.

In Fig. 96 the broad line shows the new condition of the wheels and the fine line is a repetition of Fig. 95. The velocity ratio and the normal pitch have been retained but the angles have become β_1 and β_2.

This of course is a trial and error process and is equivalent to the arithmetical process of example I.

143. Order of Procedure in Determining the Dimensions.—To obtain the approximate result for shafts at 90° the procedure is as indicated in Fig. 97, where O*n* and O*m* are lines drawn at right angles and indefinitely long, ON and OM set out in the

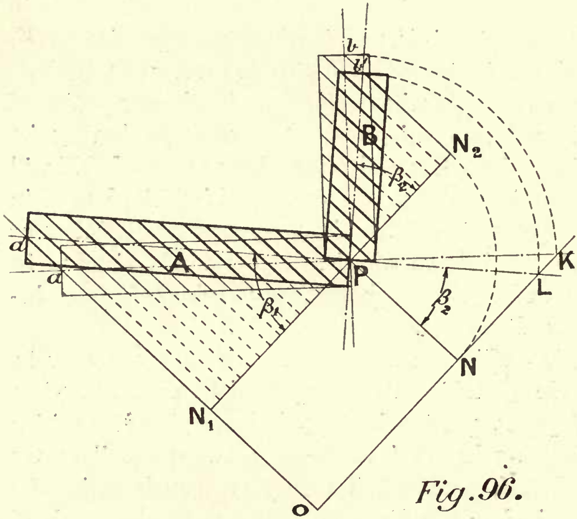


Fig. 96.

proportion of the required velocity ratio, the rectangle ONPM completed and the diagonal OP drawn in and continued. Perpendiculars from any point on OP to the lines O*n* and O*m* will have the same ratio as OM and ON. With twice the centre distance marked on the sharp edge of a piece of paper and the marks coinciding with O*n* and O*m*, as $n_1\ m_1$, the intersection P_1 with OP divides the distance in the required proportion of diameters. This is the preliminary investi-

gation. Measuring the perpendiculars P_1N_1 and P_1M_1, these should be exact multiples of the normal pitch; if they are not, other positions as n_2m_2, or n_3m_3 must be tried until the exact multiple is obtained; these multiples will then be the numbers of teeth on the respective wheels, and the screw angles will be those between the selected slant line position and the perpendiculars on On and Om from the point of intersection with OP, as $N_1P_1n_1$ and $M_1P_1m_1$ of the trial position.

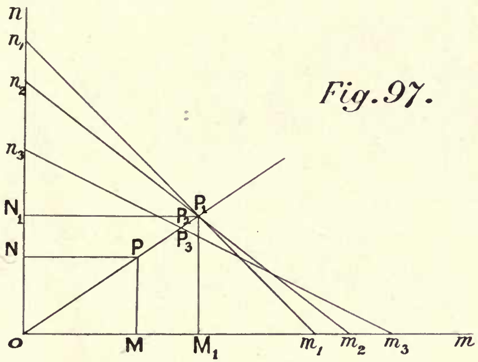


Fig. 97.

If the final solution makes one of the screw angles very small, it is advisable to reconsider the centre distance or the normal pitch to see whether they may not be altered.

If the operation be performed upon accurately engraved squared paper and a thin-edged scale be used to carry the sum of the diameters, all the drawing that is necessary is the diagonal OP, and many trials can be made in a few minutes; the perpendiculars can be read off from the squares on the paper and the division of the perpendicular dis-

tances by the normal pitch made mentally or by the aid of a slide rule; and the eye is sufficient to detect whether the angle is a good or bad one.

144. Chart for Graphical Solutions.—Fig. 98 suggests a construction for a chart upon which the preliminary surveys may be made. Upon accurately made squared paper a series of diagonals are drawn, such as OP of Fig. 97, giving different velocity ratios, a few only being shown in the figure, and the chart

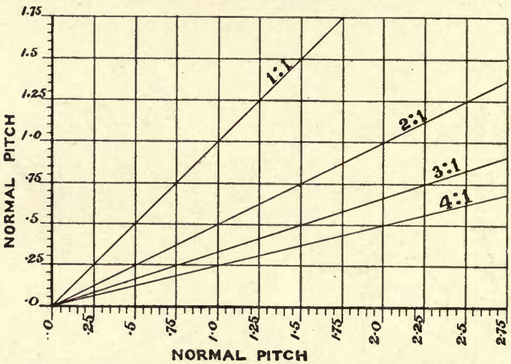


Fig. 98.

can be used in exactly the same manner as described for Fig. 97.

145. Axes at other than 90°.—Oblique axes may be solved graphically in the same way as right angled ones. Figs. 99 and 100 illustrate the method, the only difference in the diagrams being that the angle at O is the supplement of or equal to the shaft angle according as the direction of the teeth divides the obtuse, Fig. 99, or the acute, Fig. 100, angle between the shafts. The lettering is

the same as for Figs. 95 and 96 and may be understood without further description other than to point

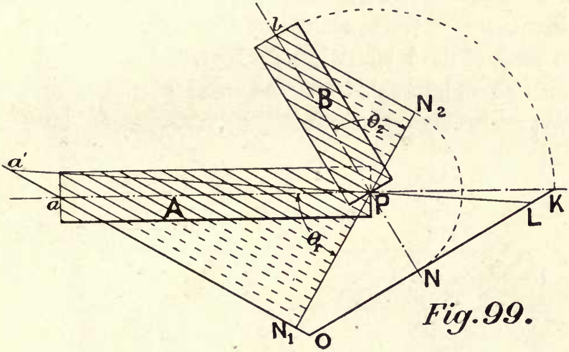


Fig. 99.

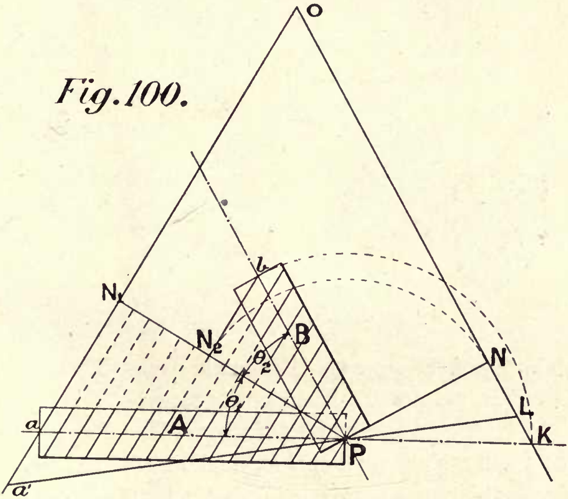


Fig. 100.

out that $PK = Pb$ must be brought in line with Pa; turning the triangle bN_2P round into this position at once fixes the directions of Oa and OK.

146. Direction of Rotation.—The hand of the screw is no guide to the direction of rotation. Figs. 101 to 104 show four different arrangements, and the direction of rotation shown by the arrows is the same in each of the lower wheels; Figs. 101 and 102 have axes at right angles but the hand of the thread is reversed; Figs. 103 and 104 have axes at 45° but the

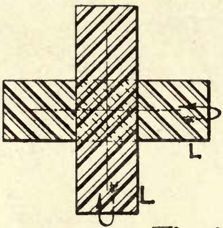


Fig. 101.

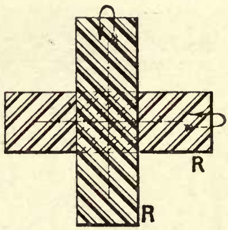


Fig. 102.

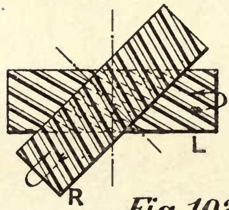


Fig. 103.

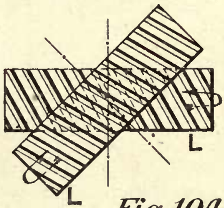


Fig. 104.

obliquity of the screw is different in each. The letters L and R indicate the hand of the screw.

147. Angle for Least Sliding.—The direction of the teeth which gives the least sliding is that which bisects the angle between the shafts, for if in Fig. 105 XX and YY are the axes of two shafts, PA and PB lines at right angles to the axes at P, PR the direction of the gearing teeth at P, and p the normal pitch,

then SQ will be the position of the gearing teeth after having passed a distance equal to p. If R be taken as the driving point during the passage, it will have travelled from R to Q at right angles to the axis XX on the one wheel, and from R to S at right angles to YY on the other, so that the total sliding is SQ.

Using symbols for the angles as in the figure

$$\frac{SQ}{RQ} = \frac{\sin \gamma}{\sin \theta_2}, \text{ and } RQ = \frac{p}{\sin \theta_1}.$$

Substituting for RQ,

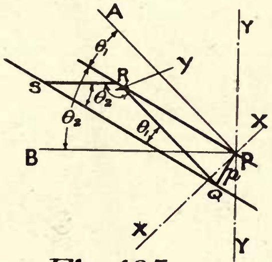


Fig.105.

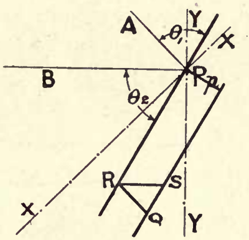


Fig.106.

$$SQ = \frac{p \sin \gamma}{\sin \theta_1 \sin \theta_2}.$$

$\theta_1 + \theta_2$ is constant being one of the angles between the shafts, then γ also is constant being the third angle of a triangle, so that $p \sin \gamma$ is constant. For SQ to have a minimum value the denominator $\sin \theta_1 \sin \theta_2$ must have a maximum value since the numerator is constant; and this occurs when $\theta_1 = \theta_2$, that is when the tooth direction bisects the angle between shafts.

Fig. 106 illustrates the case of the tooth dividing the acute angle from which it can be seen that the

sliding SQ is much less than in Fig. 105 where the tooth divides the obtuse angle.

148. Efficiency.—In Fig. 107 HH is the direction of a pair of teeth in contact at P, and the force FP tangent to the pitch cylinder and at right angles to the axis of A is driving in the direction F to P. The reaction at P, without friction, is PR_1 at right angles to HH, the direction of sliding over HH is according to the arrows shown, the one on the right applying to A, and the frictional resistance acting against the

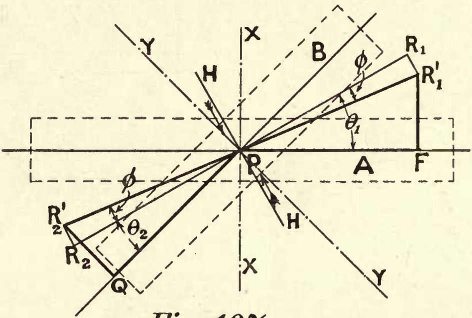


Fig. 107.

movement deflects the reaction to PR'_1 so that $R_1PR'_1 = \phi$ the angle of friction; the third force is the axial thrust which equals R'_1F parallel to XX.

The forces on the B wheel are made up of QP the resistance overcome at right angles to B's axis and tangent to its pitch cylinder, the real reaction R'_2P equal and opposite to R'_1P, and R'_2Q the axial thrust on B; the normal reaction, without friction, at P is R_2P at right angles to HH, and $R_2 PR'_2 = \phi$.

R_1PF being the angle between the normal PR_1 to the tooth and a line at right angles to A's axis it

equals the angle between the tooth and A's axis; that is R_1PF is the screw angle of A and $= \theta_1$; similarly R_2PQ is the screw angle of B and $= \theta_2$; $R'_1PF = \theta_1 - \phi$ and $R'_2PQ = \theta_2 + \phi$.

Force $FP = R'_1P \cos(\theta_1 - \phi)$ = driving force

,, $QP = R'_2P \cos(\theta_2 + \phi)$ = resisting force

$R'_1P = R'_2P$ = tooth reaction = T

then $$\frac{\text{resisting force}}{\text{driving force}} = \frac{T\cos(\theta_2 + \phi)}{T\cos(\theta_1 - \phi)} = \frac{\cos(\theta_2 + \phi)}{\cos(\theta_1 - \phi)}.$$

Since work done = force × distance, and

$$\text{efficiency} = \frac{\text{useful work done}}{\text{work expended}}$$

$$= \frac{\text{resisting force} \times \text{distance travelled}}{\text{driving force} \times \text{distance travelled}},$$

and $$\frac{\text{distance by driven point on B}}{\text{distance by driving point on A}} = \frac{\cos\theta_1}{\cos\theta_2}$$

(see paragraph 134)

$$\text{efficiency} = \frac{\cos(\theta_2 + \phi)}{\cos(\theta_1 - \phi)} \times \frac{\cos\theta_1}{\cos\theta_2} \quad . \quad \text{(IX)}$$

which reduces by trigonometrical transformation to

$$\frac{\cot\phi - \cot\theta_1}{\cot\phi + \cot\theta_2}.$$

Substituting values for θ_1 and θ_2 and assuming $\tan\phi = \cdot 05$, for axes at right angles the efficiency for θ_1 and θ_2 ranging between 40° and 50° is found to be practically constant and equal to 90 per cent.

CHAPTER XV

STRENGTH OF TEETH

149. The Tooth a Cantilever.—A spur or tooth at any time is a projecting piece from a mass of greater substance, and when considering its strength to resist a force that may come upon it it must be treated as a beam fixed at one end and free at the other, the point of fixing being the junction of the spur to the greater mass, or in the case of wheel teeth the junction of the tooth to the rim of the wheel.

With a beam of given dimensions and material, its ability or strength to resist an external force depends upon the position and direction of the applied force and the nature of the material.

150. Distribution of Load.—With straight spur teeth transmitting energy, the force coming upon any tooth in gear may be any where over the whole acting surface; under good conditions of workmanship it may be upon a line extending across the whole width of the tooth, and under poor conditions it may come all upon one corner. On wheels with helical teeth the distribution of load on each tooth is continually changing from first one end, then by a gradual growth to complete width and then a diminution to disappearance at the opposite end; the whole load however never comes on a single end unless the wheels be very roughly constructed; usually it may

be considered that the load on a tooth varies directly as the proportion of width in gear at any instant. With screw wheels a very small area only can receive the load, as is also the case with the simple worm and worm wheel, in both of which in theory point contact exists, while with the close-fitting worm, if well made, line contact may be assumed. When ordinary straight gears are machine moulded or the teeth are machine cut it is usual to assume line contact, and experience shows that the assumption is practically correct; but if the gears are pattern moulded line contact is very uncertain and the proper course is to assume point contact only.

151. Application of Beam Formula.—Assuming the tooth rectangular in section the simple beam formula may be applied, and where the tooth departs appreciably from that form allowance can be made as will subsequently appear. In Fig. 108 the small loads w make up the total load W applied along a line at the outer edge of the tooth or end of the cantilever; b is the breadth, t the depth or thickness of tooth, and l the length. The maximum bending moment caused by W is Wl and the internal moment resisting it, or moment of resistance, for the rectangular beam section is $f\frac{bt^2}{6}$, where f is the maximum stress in the material reckoned per square inch of section in the same units as W.

Equating the two moments

$$Wl = f\frac{bt^2}{6}$$

from which

$$f = \frac{6Wl}{bt^2} \quad . \quad . \quad . \quad (X)$$

From X it can be seen that the maximum stress varies directly as the length l and inversely as the square of the depth of the beam t (or the thickness of tooth).

Fig. 109 is a bending moment diagram for the tooth of Fig. 108. The vertical lines on the end indicate the manner in which the stress varies from nothing at the free end to a maximum at the fixing.

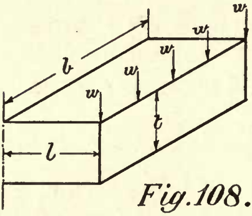


Fig. 108.

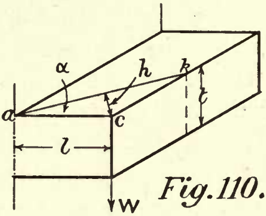


Fig. 110.

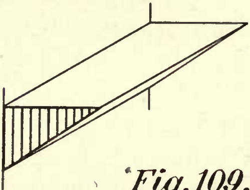


Fig. 109.

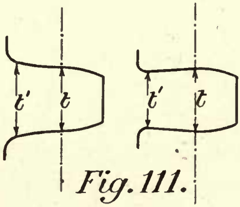


Fig. 111.

152. Load on a Corner.—In Fig. 110 the total load W is upon the corner and the tooth may break across some diagonal line such as ak; the perpendicular distance of the corner c from ak equals h and the angle $cak = \alpha$, then $h = ac \sin \alpha = l \sin \alpha$.

Treating the piece ack as a cantilever loaded at c, the bending moment is Wh and the section resisting the moment is the rectangle under ak which $= ak \times t$; ak being the breadth and t the depth, and

the moment of resistance is $f\,\dfrac{ak \times t^2}{6}$. Equating this to the bending moment,

$$Wh = f\frac{ak \times t^2}{6},$$

and since $ak = \dfrac{ac}{\cos\alpha} = \dfrac{l}{\cos\alpha}$ and $h = l\sin\alpha$

by substitution $\qquad Wl\sin\alpha = f\dfrac{lt^2}{6\cos\alpha}$

and $\qquad f = \dfrac{6Wl\sin\alpha\cos\alpha}{lt^2};$

but $2\sin\alpha\cos\alpha = \sin 2\alpha$, therefore

$$f = \frac{6Wl\sin 2\alpha}{2lt^2} \quad . \quad . \quad \text{(XI)}$$

When $\alpha = 45°$, $2\alpha = 90°$, and $\sin 2\alpha = 1$, which is the maximum value $\sin 2\alpha$ can have and the corresponding value of f will be a maximum, then

$$f = \frac{6Wl}{2lt^2} \quad . \quad . \quad . \quad \text{(XII)}$$

The corner may be expected to break off at 45° if it break at all.

For equal stress f in the two cases equate X and XII, then $\qquad \dfrac{6Wl}{bt^2} = \dfrac{6Wl}{2lt^2}$

from which $b = 2l$.

Thus twice the total height or length of the tooth may be taken as the minimum width for pattern moulded cast gears; with this proportion the tooth is equally strong by either method of loading. With total height of tooth = ·7 pitch, width = 2 × ·7 = 1·4 pitch or $1\frac{1}{2}$ pitch approximately.

153. Some Conditions which Influence the Strength.—The fact that usually more than one pair

of teeth are in gear at any moment may ameliorate the stress conditions. If when two pairs are in gear reasonably good contact is obtained then the load will be borne partly by each. On reference to paragraphs 19 to 21 it will be seen that except in the case of the teeth of both wheels being numerous, the arc of contact does not extend to twice the pitch, and consequently there is a short distance near the middle of the arc where only one pair is in gear, but here the point of application of the load (contact of the teeth) is near the pitch point and the effective length of the cantilever is thereby reduced to nearly one half, and with it the maximum stress in the same proportion.

On the assumption that each tooth in gear has proper contact it is not unreasonable to suppose that each takes its share of the load, so that when two pairs are in gear each receives half; and as the proportion of stress when contact is near the pitch point, and only one tooth in gear, is nearly half that when contact and whole load is on one outer edge only, it is safe to reckon that at no time is the tooth so badly off as to have the whole load on the outer edge; with arc of contact equal to twice the circular pitch, or more, the tooth is still better off. It would seem fair then to reckon the load on a tooth equal to

$$W \frac{\text{circular pitch}}{\text{arc of contact}}$$

and this load applied at the outer edge.

If the teeth are reasonably well made in the first instance yet contact occurs at a corner, after the teeth have worn to a bearing the contact will have extended and thereby improved the stress conditions.

Another item in favour of the tooth is the fillet where the root joins the wheel rim. In the length l of Figs. 108 and 110 the clearance was included but as part of this is used for the fillet the effective length is rather less than height plus depth of tooth.

154. Weak Roots.—The profile of the tooth may influence the strength either way as calculated upon the basis of a uniform rectangular section, for whereas the practice is to assume the depth of the cantilever the same as the thickness of the tooth at the pitch line, in the case of radial and under cut flanks the thickness near the base is less than at the pitch line, and the base is the weakest part, for the maximum bending moment occurs there. In such cases the thinnest part of the tooth should be taken as the thickness; or working out the stress in the ordinary way the result should be multiplied by $\frac{t^2}{t_1^{\,2}}$ where t is the pitch line thickness and t_1 the thinnest section below the pitch line. Any reduction in thickness above the pitch line does not affect the strength as the stress there is small.

155. Strong Roots.—In many cases the profile broadens towards the base and thereby adds strength, and if the weight of the total construction being designed is to be kept as low as possible, the designer should make use of this advantage and calculate upon the actual tooth section. If the thickness of the tooth just at the springing off of the fillet circle be taken as the depth of the cantilever all is safe. By taking advantage of this a reduction of pitch may result and as other dimensions, including the framework of the machine, follow suit, a con-

siderable saving of material, labour, and cost of production may be effected.

The calculations for the thick base can be made in the same way as for the thin one, namely by using the multiplier $\frac{t^2}{t_1^2}$, but in this case t_1 being the thickness next the fillet it is greater than t (see Fig. 111).

156. Speed Effects.—Speed of pitch line also affects the strength by increasing the violence of the shock as each tooth enters into gear, or shock received from other parts of the machinery. Provision for this is made in all-metal wheels by the selection of a suitable value for f.

157. Formulæ for Pitch.—To arrive at the pitch from the formulæ X and XII, for stress, it is only necessary to put ld and b in terms of pitch and substitute the allowable f.

With $t = {\cdot}48p''$, $l = {\cdot}7p''$, and $b = 3p''$, where $p =$ pitch.

$$\text{X becomes } f = \frac{6\text{ W} \times {\cdot}7\,p}{3p \times ({\cdot}48p)^2} = \frac{4{\cdot}2\text{ W}p}{{\cdot}6912p^3} = 6{\cdot}076\,\frac{\text{W}}{p^2}$$

$$\text{and } p = 2{\cdot}46\sqrt{\frac{\text{W}}{f}}; \qquad . \qquad . \qquad \text{(XIII)}$$

when W is known and the limit of f is fixed p can be found.

$$\text{XII becomes } f = \frac{6\text{ W} \times {\cdot}7p}{2 \times {\cdot}7p \times ({\cdot}48p)^2} = \frac{3\text{ W}}{{\cdot}2304p^2}$$

$$\text{and } p = 3{\cdot}6\sqrt{\frac{\text{W}}{f}} \qquad . \qquad . \qquad \text{(XIV)}$$

For machine cut gears having stub teeth; $t = {\cdot}5p''$, $l = {\cdot}55p''$, and $b = 4p''$, from X

$$f = \frac{6\text{W} \times {\cdot}55p}{4p \times ({\cdot}5p)^2} = \frac{3{\cdot}3\text{W}}{p^2},$$

$$\text{and } p = 1{\cdot}82\sqrt{\frac{W}{f}} \quad . \quad . \quad \text{(XV)}$$

158. Pitch Line Pressure on Teeth.—The value of W at the pitch line can be found if pitch line velocity and the horse power transmitted be known.

Since work done = force × distance travelled,

and work per min. = H.P. × 33,000 ft. lb.,

and distance travelled per min. = revs. per min. × pitch circumference = N × 2πR,

where N = revs. per min., and R = pitch radius in feet; then

$$\text{force W, or press. on the tooth} = \frac{\text{H.P.} \times 33{,}000}{2\pi \text{RN}} \text{ lb.};$$

and this may be assumed to come wholly on the corner of a pattern moulded wheel tooth, and in machine moulded and machine cut teeth may be distributed according to the number of teeth in gear as pointed out in paragraph 153.

159. Effect of Number of Teeth and Wheel Diameter on the Tooth Pressure.—The actual force on a tooth is rather different from the above owing to the fact that contact is not always at the pitch point, for which place W was calculated. Pressure will come at the outer edge of the tooth at first and last contact and the direction will be normal to the surfaces with a deviation due to friction.

In Fig. 112 which shows a pair of teeth having just found contact, F (= W of paragraphs 157 and 158) is the tangential force at P necessary for transmitting the power; at the contact point C this force is reduced in the proportion $\frac{\text{OP}}{\text{OP}'}$ by the principle of the lever, O being the centre of driven rotation. CQ

represents to scale the force at C at right angles to OP, the reaction at C is in the direction C to the pitch point P and equals CR, and QR is the thrust on the bearing in direction C to O. Modified by friction, the reaction is in direction CR_1, where $RCR_1 = \phi$ the angle of friction; then CR_1 is the actual force on the tooth.

If CR_1 be resolved in direction CQ_1 at right angles to OM the centre line of the tooth and Q_1R_1 parallel to OM, Q_1R_1 represents the force tending to crush

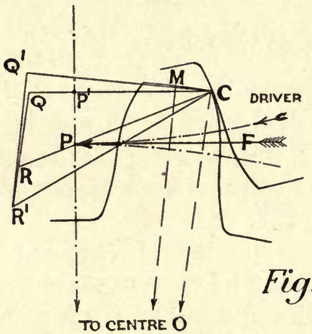


Fig. 112.

the tooth and may be neglected, for the materials in common use have ample crushing strength, and CQ_1 represents the force tending to break the tooth transversely and is the one to be considered. From the diagram it is evident that CQ differs but little from CQ_1, and CQ as already pointed out is less than W in the proportion given above. Where the teeth are numerous this reduction of W is small and may usually be neglected; for example in a 30-toothed wheel circular pitch $= \frac{2\pi R}{30} = \frac{6{\cdot}28}{30}R$, and with height

of tooth = ·3 pitch = $\frac{\cdot 3 \times 6\cdot 28}{30}R = \frac{1}{17}R$ or 6 per cent very nearly. As the number of teeth increases the reduction becomes less and less, but as the number diminishes it becomes greater, and should be accounted for in a close design.

The reduction from W to CQ_1 in the case of low-numbered pinions is a set-off against under cutting but must not be taken to equal it; each must be properly accounted for in a design where the strength is cut fine.

160. Allowable Stress.—The allowable stress f is a quantity that depends upon the kind and quality of the material used and the possibility of shock when running; the former conditions can be determined accurately but the latter must always be a matter of judgment within fairly wide limits.

Some authorities on wheel teeth, notably Prof. Reuleaux and Mr. Wilfred Lewis, have given tables of allowable stresses under different conditions of running. Two tables follow, in each of which f is in pounds per square inch of section. That of Mr. Lewis is probably the more reliable, and is widely used in this country and America.

VALUES OF f FOR TRANSMISSION GEARS (REULEAUX)

Pitch line velocity in f.p.m.	100	200	400	600	800	1000	1500	2000	2500
Cast iron f =	4240	4060	3744	3473	3238	3034	2620	2302	2068
Steel f =	14,112	13,020	12,467	11,565	10,782	10,103	8725	7665	6886
Wood f =	2544	2436	2246	2083	1943	1820	1572	1381	1240

VALUES OF f FOR TRANSMISSION GEARS (LEWIS)

Pitch line velocity in f.p.m.	100	200	300	600	900	1200	1800	2400
Cast iron f =	8000	6000	4800	4000	3000	2400	2000	1700
Steel f =	20,000	15,000	12,000	10,000	7500	6000	5000	4300

161. Cut Helical Teeth: Rules for Design.—The teeth of helical wheels do not lend themselves to quite such simple theoretical treatment as those of straight spurs, but experience has resulted in producing some empirical rules which may be safely used for these gears when machine cut.

The substance of the following rules and remarks has been abstracted from a paper read before the American Society of Mechanical Engineers by Mr. P. C. Day.

The teeth are assumed to be straight across and the whole of the load to be on one tooth. The shearing stress K is reckoned by distributing the load W over the pitch line section, and velocities V are reckoned at the pitch line. With other symbols as in paragraph 157

$$V = 2\pi RN,$$

$$W = \frac{\text{h.p.} \times 33000}{V},$$

$$W = \frac{pbK}{2} \text{ (thickness} = \cdot 5p).$$

$$b = \frac{2W}{pK},$$

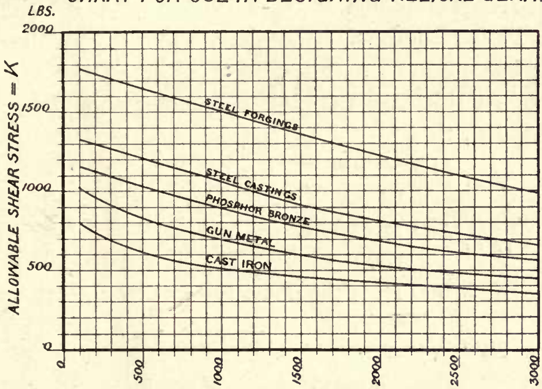
$W = 3p^2K$ (for normal gears of moderate size, $b = 6p$),

$p = \sqrt{\dfrac{W}{3K}}$, with high ratios use $b = Qp$ (Q = ratio of the gearing) up to a maximum of $b = 10p$.

and $p = \sqrt{\dfrac{2{\cdot}5W}{QK}}$.

These rules are to be used in connexion with the chart below.

CHART FOR USE IN DESIGNING HELICAL GEARS



162. Worked Examples.—Generally the horse power and the revolutions are known quantities, and often centre distance or diameters are fixed, in which case there is no choice of dimensions; when, however, diameters are not fixed, experience or judgment must decide what is suitable.

Velocities up to 2000 ft. per min. at the pitch line may be safely used. Pinions overhanging a

bearing should be larger than when supported on both sides. Cast iron is preferable for gears of large diameter except when the tooth pressure is great, in which case steel castings should be used. Pinions should be of forged steel containing ·4 to ·5 per cent carbon.

As an example of choice of dimensions take the case of a pump requiring 150 H.P. at 50 r.p.m. and driven from a motor running at 500 r.p.m. with a shaft end of $4\frac{1}{2}''$ diameter.

Three distinct cases present themselves. A—Overhanging pinion. B—With an outer bearing. C—Pinion solid with the shaft and with an outer bearing.

A. In this case the pinion diameter should be at least 10″, then

$$V = \frac{2\pi 5 \times 500}{12} = 1300 \text{ ft. per min.}$$

$$W = \frac{150 \times 33000}{1300} = 3800 \text{ lb.}$$

$$K = 500 \text{ from the chart for cast-iron gears}$$

$$p = \sqrt{\frac{3800}{3 \times 500}} = \sqrt{2{\cdot}53} = 1{\cdot}59 \text{ in.}$$

$$b = \frac{2 \times 3800}{1{\cdot}59 \times 500} = 9{\cdot}5 \text{ in.}$$

B. This case may be reckoned for a cast-iron or cast-steel gear.

B I. Cast-iron gear with pinion $7\frac{1}{2}''$ diameter, then

$$V = \frac{2\pi 3{\cdot}5 \times 500}{12} = 975 \text{ ft. per min.}$$

$$W = \frac{150 \times 33000}{975} = 5100 \text{ lb.}$$

$$K = 530 \text{ from the chart.}$$

$$p = \sqrt{\frac{5100}{3 \times 530}} = \sqrt{3 \cdot 2} = 1 \cdot 79 \text{ in.}$$

$$b = \frac{2 \times 5100}{1 \cdot 79 \times 530} = 10 \cdot 8 \text{ in.}$$

B II. Cast-steel gear with pinion $7\frac{1}{2}''$ diameter, then

$V = 975$ ft. per min. as in B I

$W = 5100$ lb. as in B I

$K = 1060$ from the chart for cast steel

$$p = \sqrt{\frac{5100}{3 \times 1060}} = \sqrt{1 \cdot 6} = 1 \cdot 265 \text{ in.}$$

$$b = \frac{2 \times 5100}{1 \cdot 265 \times 1060} = 7 \cdot 6 \text{ in.}$$

C. In this case a $5''$ pinion diameter may be used with a cast-steel gear wheel, then

$$V = \frac{2\pi 2 \cdot 5 \times 500}{12} = 650 \text{ ft. per min.}$$

$$W = \frac{150 \times 33000}{650} = 7600 \text{ lb.}$$

$K = 1150$ from the chart

$$p = \sqrt{\frac{7600}{3 \times 1150}} = \sqrt{2 \cdot 2} = 1 \cdot 48 \text{ in.}$$

$$b = \frac{2 \times 7600}{1 \cdot 48 \times 1150} = 9 \text{ in.}$$

The nearest convenient pitches and widths to these results would be used.

When calculating the tooth pressure the average working load is sufficiently near unless the maximum is very much greater than the average, in which event the mean between the average and the maximum may be taken. Exceptions to this method occur in rolling mills, where the teeth may be subjected to considerable overload, and steam turbines where

the velocity is very high; in the former case some estimate of the possible overload must be made and the teeth designed to be strong enough to meet it, and in the latter case extreme width and very fine pitch used.

163. Shrouding.—To give some extra strength to cast teeth they are sometimes "shrouded," that is a flange or plate of thickness about five-eighth pitch is cast at the sides of the wheel, solid with it and the teeth, which gives an additional support to the ends of the teeth. When carried up to the pitch line it is "half shrouded," when on one side only and taken to the top it is "single shrouded" and when on both sides to the top it is "double shrouded". Both wheels which gear together cannot have the shrouding to the tops on both sides, they must be either both half shrouded or both single shrouded, with the shrouding arranged to be on opposite sides when in mesh. The most common practice is to shroud pinions only and then on both sides; single shrouding is by some considered useless as it leaves one end of the teeth free and unprotected.

164. Deflection of Teeth.—When a force or load comes upon a tooth some deflection must occur, and according to Hooke's law of stress and strain the deflection varies directly as the load. Applying this law to the teeth of wheels, those that are in gear at any instant are deflected while those not in gear being unstressed are undeflected, with the result that the tooth just coming into gear is out of pitch with the one next it in gear; the two wheels are oppositely affected, one has its pitch shortened and the other has it lengthened, as can be seen from Fig. 113 where

the dotted lines shew the undeflected teeth and the full lines the running condition. The conjugate surfaces are not in their proper relative positions at first contact, and the teeth come together with impact which becomes worse as the velocity increases; hence the noise when running even with the best of spur gears.

To *absorb* the shock, raw hide or wood is used for the teeth of one wheel when running at high velocities. To *avoid* the shock as much as possible the scheme is to ease off the faces near the tops; this allows the

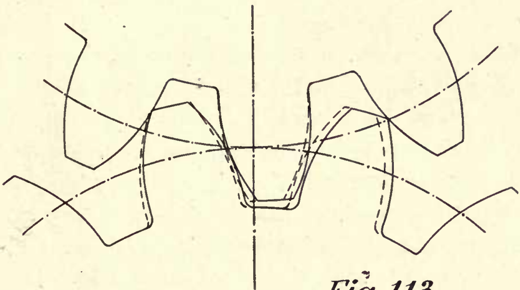


Fig. 113.

tooth to take up its deflection slowly with beneficial results to the smoothness of running and the stability of the tooth. The easing off of the tops for this purpose must not be confused with the easing off of involute teeth which, in order to make them of standard height, are carried beyond the point of interference. Helical teeth also reduce or avoid shock by taking up deflection slowly.

165. Strength of Bevel Wheel Teeth.—The strength of bevel wheels is not so simple a matter to decide as that of straight spurs because of the changing section from the large to the small end. To

calculate from the dimensions at the large end and neglect the diminishing section is obviously incorrect; while to use the dimensions of the small end only seems to be neglecting the value of material advantageously situated. The section of mean strength value between the two ends is at the mean diameter, and may be found as follows: the proportionate sizes of the teeth will be directly as their distance from the apex of the pitch cone, then assuming the large end $\frac{3}{2}$ times the small end, and remembering that the strength varies as $\frac{t^2}{l}$, these symbols having the same meaning as in paragraph 151 the comparison is

$$\frac{\text{strength at large end}}{\text{strength at small end}} = \frac{t^2}{l} \div \frac{(\frac{2}{3}t)^2}{\frac{2}{3}l} = \frac{t^2}{l}\,\frac{18l}{12t^2} = \frac{3}{2},$$

or directly as the distance from the apex. The position of mean strength then is half way across the width of the wheel.

To use the dimensions at this position would be to assume contact for the whole width, which is only possible with the most perfect fitting; and considering the fact that bevel wheels, in addition to being themselves well made, must have their shafts set perfectly at the angle of design in order to get line contact, it is highly probable that the pressure comes on the teeth chiefly at one end, which end depending upon the erection.

Since the chances are equal unless bias towards the large end can be ensured, the proper course is to calculate upon the section at the small end.

CHAPTER XVI

DURABILITY

166. Durability.—The durability of a tooth is nearly as important as the actual strength. Figures giving the amount of wear under different conditions are not obtainable, and what provision is made rests entirely upon the judgment of the designer. Some conditions which influence the durability however are known.

167. Amount and Speed of Sliding Contact.—The chief factor which causes wear is the rubbing or sliding of the working surfaces of the teeth, and assuming a constant intensity of pressure the amount of wear may be taken to be directly as the amount of sliding in any given time. This amount of sliding for one period of meshing is equal to the difference in length of the working face and working flank reckoned separately on approach and recess, and is greater when contact is near the outer edge of the tooth than when near the pitch point; this fact can be verified by actually drawing the teeth in successive positions, or by direct observation, and measuring the sliding; or it can be arrived at mathematically. The following investigation is useful in this connexion.

Using Fig. 23 for reference: When the two wheels A and B rotate with angular velocities ω_1

and ω_2 respectively, two coincident points of contact, as n_1, on each wheel are passing each other with a velocity which is compounded of the motion of both wheels. If the B wheel be supposed stationary while A journeyed round it in the same time as B would have taken to revolve once, the *relative* movements of all the points on A and B would be the same with regard to direction and speed as if the wheels had rotated together on their fixed axes; and the rotations of A about its own axis would be $1 + \frac{\text{diam. of B}}{\text{diam. of A}}$ (see epicyclic trains, paragraph 174), which will give an angular velocity equal to the sum of ω_1 and ω_2.

Assuming then that A rolls round B, n_1 on B's tooth is stationary while n_1 on A's tooth turns about P, the contact of the pitch lines, with the angular velocity of the wheel A, viz. $\omega_1 + \omega_2$; Pn_1 will be the instantaneous radius and $(\omega_1 + \omega_2)\ Pn_1$ the linear velocity of sliding at n_1. At n_2 the sliding velocity will be $(\omega_1 + \omega_2)\ Pn_2$, and so on. Thus the speed of sliding is proportional to the distance of the contact point from the pitch point, and is therefore quicker near the outer edge of the tooth than near the middle where it is zero. Since equal parts of the arc of contact are passed through in equal times, those parts which are nearer to the ends slide over the greater distance, consequently the loss of energy is greater there and the abrasion or wear of the tooth is greater.

168. To keep down the Wear.—Distinctly then to keep down the wear the contact should be brought near to the pitch point, but this can only be done by

reducing the height and depth of the tooth or by reducing the pitch without altering the proportions of the tooth. Both methods are used; the former shortens the arc of contact and thereby lessens the number of teeth in gear but strengthens the tooth, the latter also shortens the tooth and the arc of contact, but the pitch being less the number of teeth in gear is equal or more (see paragraphs 19 to 21).

169. Increasing the Width.—From equation X $f = \frac{6Wl}{bt^2}$, and if for l and t their values in terms of pitch be inserted, and then for all the constant quantities the symbol C be put $f = \frac{C}{bp}$ or $f \propto \frac{1}{bp}$, from which it is clear that any change in pitch if accompanied by an opposite and proportionate change in the width does not affect the value of f. A reduction of p together with an increase of b not only retains the old strength of the tooth and gives the advantage of reduced sliding, but also introduces a reduction in the intensity of pressure at any place by the wider distribution of the load, and thereby a still further reduction of wear. Widths up to 4 and $4\frac{1}{2}p$ are used in cases of wheels subjected to hard continuous running. For greater widths than these the straight spur is unsuitable.

170. Short Teeth.—Short teeth have an advantage over long ones in that the obliquity of thrust is less. With regard to sliding the shortened teeth and those of reduced pitch are equally well off if made involute with the same obliquity or if made cycloidal with the same rolling circles, the profiles of the reduced or shortened teeth being parts only of the larger ones.

If it be thought undesirous to shorten the total length of tooth, a better running condition may be obtained by throwing the action more into the arc of recess. The advantage of this may be seen by reference to Fig. 112, which will serve to illustrate, although not accurately, the diagram being drawn for approach. During recess the direction of sliding is opposite to that during approach, in which case the angle RCR_1 must be set out on the other side of CR and thereby reducing QR_1, or eliminating it altogether, and at times placing it on the other side of Q; the result of which is that the thrust on the bearings alternate a little only on either side of zero, with smooth running as a result.

CHAPTER XVII

TRAINS OF WHEELS

171. Trains of Wheels.—Thus far angular velocity ratio considered has always been that between *two* wheels which gear together. It may be of interest to some readers to go into the matter of the velocity ratio obtained by using *trains of wheels* containing more than two.

Instead of the term "angular velocity" its equivalent "revolutions per minute" or "revolutions in the same time" will sometimes be used; and the revolutions of the last wheel of a train compared with those of the first in the same time will be known as the velocity ratio of the train.

172. Simple Train.—A series of wheels gearing together in the manner of Fig. 114 is known as a *simple train,* and all between A and L are called idle wheels in so far as they affect the velocity ratio; their chief function being merely to change direction of rotation or to make up distance. The velocity ratio of a simple train will always be inversely as the diameters of the first and last wheels, and will, in consequence, be no different from that of the first and last gearing directly into one another if direction of rotation be neglected.

173. Compound Train.—A series of wheels arranged as in Fig. 115 is known as a *compound train;*

B and C are rigidly fixed to the same spindle so that they rotating together make the same revolutions in any given time, and D and E are under similar conditions. The train as it appears in the sketch is compounded of three simple trains of two wheels each, namely, A into B, C into D, and E into L. Assuming

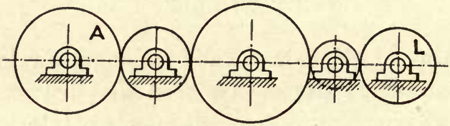


Fig. 114.

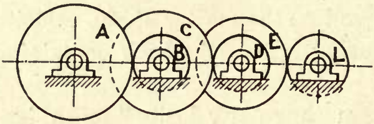


Fig. 115.

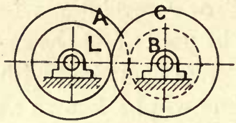


Fig. 116.

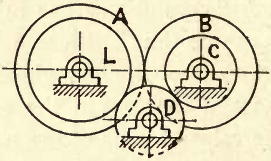


Fig. 117.

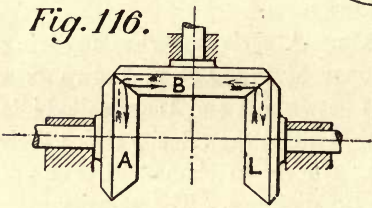


Fig. 118.

A as the source of motion, the rotations imparted to B are given to C, and C turns D while E simultaneously receives the same as D, and E transmits the motion on to L.

From each simple train of two wheels :—

$$\frac{\text{revs. of B}}{\text{revs. of A}} = \frac{\text{diam. A}}{\text{diam. B}}; \quad \frac{\text{revs. of D}}{\text{revs. of C}} = \frac{\text{diam. C}}{\text{diam. D}};$$

$$\frac{\text{revs. of L}}{\text{revs. of E}} = \frac{\text{diam. E}}{\text{diam. L}};$$

and on compounding
revs. of C = revs. of B and revs. of E = revs. of D; substituting the values of these last two equations and retaining the letter only as representing the corresponding diameter,

$$\text{revs. of C} = \quad \text{revs. of B} = \frac{A}{B}\,\text{revs. of A},$$

$$\text{revs. of E} = \quad \text{revs. of D} = \frac{C}{D}\,\text{revs. of C},$$

$$= \frac{C}{D} \times \frac{A}{B}\,\text{revs. of A}$$

$$\text{revs. of L} = \frac{E}{L}\,\text{revs. of E} = \frac{E}{L} \times \frac{C}{D} \times \frac{A}{B}\,\text{revs. of A},$$

then velocity ratio $= \dfrac{\text{revs. of L}}{\text{revs. of A}} = \dfrac{E}{L} \times \dfrac{C}{D} \times \dfrac{A}{B}$,

which is the continued product of the velocity ratios of the individual pairs.

Expressed in words it is

$$\frac{\text{revs. of last wheel}}{\text{revs. of first wheel}} =$$

$$\frac{\text{product of diams. or nos. of teeth of drivers}}{\text{product of diams. or nos. of teeth of followers}}. \qquad \text{(XVI)}$$

174. Reverted Train.—A train which doubles back on itself as Fig. 116, B and C being rigidly connected but A and L capable of independent rotation, is called a *reverted train,* and the velocity ratio is calculated in the same manner as a compound train; that is velocity ratio $= \dfrac{A}{B} \times \dfrac{C}{L}$.

In the arrangement drawn the sum of diameters A

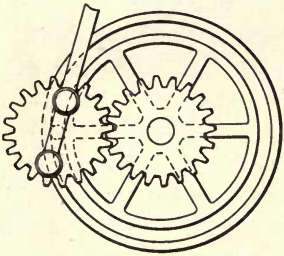
and B equals the sum of C and L, for the centre distance is half the sum of the diameters of both gearing pairs; if however the velocity ratio required and the wheels at disposal will not permit of this, then a third spindle can be introduced carrying two independent idle wheels, and if this should produce the wrong direction of rotation a single idle wheel must come in as in Fig. 117.

175. Bevel Wheel Trains.—For bevel wheel trains velocity ratios are obtained in the same manner as above, but direction of rotation must be carefully watched for with three such wheels in a train, the third rotates in the opposite direction to the first, not in the same direction as with ordinary spur wheels (see Fig. 118).

176. Epicyclic Trains.—When a train of wheels is mounted upon an arm or other framework capable of rotation in such a manner that the rotation of the arm can impart rotation to the wheels the train is known as an *epicyclic train.*

177. Sun and Planet Wheels.—The simplest epicyclic train is found in the sun and planet wheels as applied by James Watt to his early beam engines. Two equal wheels were arranged as in Fig. 119, one being keyed to the shaft and the other fixed to the connecting rod from the beam, and the two linked together so that they could not fall out of mesh. As the beam oscillated up and down the wheel on the rod, always retaining approximately the same vertical position, moved round the one on the shaft and caused it to make two rotations to one complete oscillation of the beam. A study of Fig. 120, A, B, C, D, and E will make it clear that the 2 to 1 ratio results,

for the numbers 1, 2, 3, and 4 on each wheel must gear together. The outer wheel or planet always retaining its upright position although moving round the inner one makes no rotations on its axis.



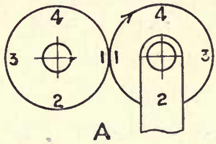


Fig. 119.

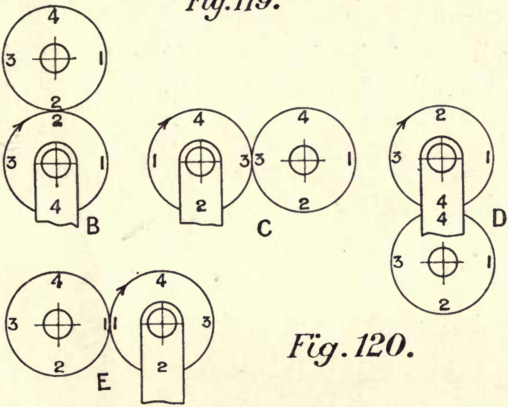


Fig. 120.

The next simplest train contains three wheels as in Fig. 122, and the arrangement is found in rope-making machinery in a triple or quadruple set.

178. To Find the Effect of the Arm, Summational Method.—In the case of Fig. 122 the wheel A is fixed so as not to rotate the other two being free on

their spindles, and the arm being carried round once the last wheel L will make $1 - \frac{A}{L}$ turns for one of the arm, A and L representing the diameters or numbers of teeth on the wheels. That this is so can be found by first locking the three wheels to the arm and turning the arm round once, each wheel will during the operation make one complete turn on its axis, not on the spindle on which it is mounted for that also will turn with the arm; then unlocking the wheels and checking the arm from rotating, and neutralizing the rotation of A by turning it backwards and allowing it to turn the other two as if all were upon a fixed bracket; the rotations given to L by this reversal will be $\frac{A}{L}$ in the same direction as A, which was opposite to that in which the arm first moved. The total turns of A will now have been

$$+1 \text{ and } -1 = 0, \text{ and of L, } +1 \text{ and } -\frac{A}{L} = 1 - \frac{A}{L}.$$

When considering the turns of L the middle wheel B was neglected. To find B's turns L is neglected, and with the same operation as before, on reversing A to bring it back to zero rotations, B is turned $\frac{A}{B}$ times opposite to A, that is in the same direction as the arm; the total rotations of B then become $1 + \frac{A}{B}$ in the same direction as the arm.

In this manner the rotations of any wheel of an epicyclic train, due to the movement of the arm, can be arrived at, and if any one of the wheels receive other rotations from an independent source, the effect

upon the remaining wheels is found by treating the arm as fixed while the effect is being produced; adding the two results gives the total rotations. This method which may be called the summational method becomes troublesome in the more complex arrangements.

179. Relative Rotations, Wheel and Arm.—Problems upon epicyclic trains are capable of solution by the very simple equation for velocity ratio given in paragraph 173, if in that equation revolutions be reckoned relatively to the arm; for when revolutions are so considered the calculation is the same as if the arm were a fixed bracket.

In Fig. 121, if the arm turn once clockwise while the wheel A remains stationary the *relative movement* of A with respect to the arm is once anti-clockwise, and if A has independent rotations given to it in either direction while the arm turns once as before, the relative rotations will be the independent ones together with one anti-clockwise; for example: calling clockwise positive and anti-clockwise negative, if A be given 5 rotations negative while the arm goes once positive, the relative rotations will be $-5-1$ or -6; if the independent turns of A be 5 positive while the arm turns once positive, then the total relative turns will be $+5-1=+4$.

Similarly with any wheel of an epicyclic train, whatever the actual turns on its axis be, when reckoned relatively to the arm, negative one must be added for each positive turn of the arm.

180. Formula for Solution of Problems.—Symbolizing all the quantities thus :—

$$\frac{\text{product of drivers}}{\text{product of followers}} = \left\{ \begin{array}{c} \text{velocity ratio or} \\ \text{value of the train} \end{array} \right\} = e$$

$$\left.\begin{array}{ll} \text{revs. of first wheel} & = m \\ \text{revs. of last wheel} & = n \\ \text{revs. of the arm} & = a \\ \text{revs. of first wheel} & \\ \quad\text{relatively to the arm} & = m - a \\ \text{revs. of last wheel} & \\ \quad\text{relatively to the arm} & = n - a \end{array}\right\} \begin{array}{l}\text{all in the} \\ \text{same time.}\end{array}$$

Equation XVI may be stated as

$$\frac{\text{product of drivers}}{\text{product of followers}}$$

$$= \frac{\text{revs. of last wheel relatively to the arm}}{\text{revs. of first wheel relatively to the arm}}, \qquad \text{(XVII)}$$

or in algebraic symbols

$$\pm e = \frac{n - a}{m - a};$$

the positive or negative value of e being decided in the following manner:—

181. The Sign of the Value of the Train (or Velocity Ratio).—With the arm fixed and the wheels pulled round, if the first and last wheels turn in the *same* direction the *positive* sign is used, if in the *opposite* direction the *negative* sign.

This rule will apply in every case, and it is better not to regard number of axes at all in considering direction of rotation, or confusion may arise with reverted trains and annular trains and bevel wheel trains.

It does not matter what the mechanical construction of the arm is like, so long as the axes of some of the wheels rotate about the axes of one or more of the others, the wheels being in mesh the train is

epicyclic, and rotations about their own axis are given to the wheels carried by the arm.

182. Possible Arrangements.—Some of the possible arrangements are illustrated in

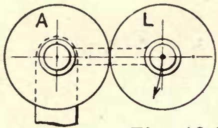


Fig. 121.

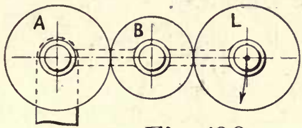


Fig. 122.

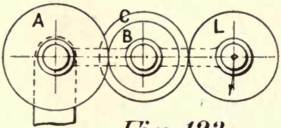


Fig. 123.

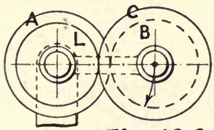


Fig. 124.

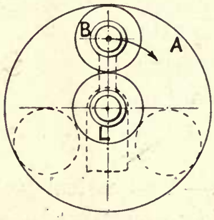


Fig. 125.

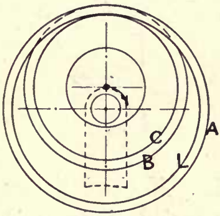


Fig. 126.

Fig. 121	which is a	2-wheel	simple	epicyclic train,
,, 122	,,	3-wheel	,,	,,
,, 123	,,	4-wheel compound		,,
,, 124	,,	4-wheel	reverted	,,
,, 125	,,	3-wheel	annular	,,
,, 126	,,	4-wheel	,,	,,
,, 127	,,	3-wheel	bevel	,,
,, 128	,,	,,	,,	,,
,, 129	,,	5-wheel double		,,
,, 130	,,	,,	,, bevel	,,

In Fig. 125 the dotted circles are duplications of B and are inserted to obtain balance of pressure; they are all carried on one piece which constitutes the arm. Fig. 129 may with advantage be treated the same way.

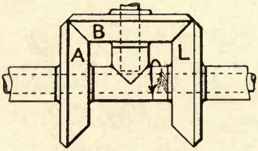


Fig. 127.

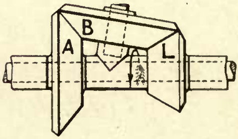


Fig. 128.

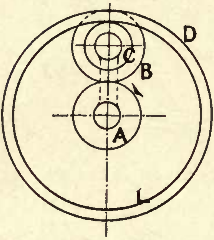


Fig. 129.

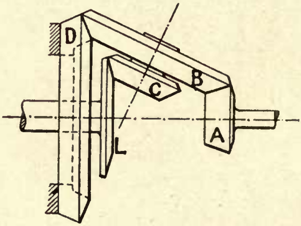


Fig. 130.

183. Worked Examples.—A numerical example upon each train is worked below, and A has been considered the first wheel and L the last.

Fig. 121.—If A has 30 teeth and is fixed, L 20, and the arm makes one revolution positive, what are the revolutions of L?

Applying the rule for the sign of e it is found negative and

$$-e = \frac{n-a}{m-a}; \quad -\frac{30}{20} = \frac{n-1}{0-1}; \quad 30 = 20\,n - 20;$$

$$n = +\tfrac{5}{2} \text{ turns};$$

that is L rotates about its axis $2\frac{1}{2}$ turns for one rotation of the arm, and in the same direction.

Fig. 122.—If A has 30 teeth and is fixed, B 15, L 30, and the arm makes one rotation positive, what are the revolutions of L?

The sign of e will be positive and the size of B is immaterial, then

$$e = \frac{n - a}{m - a}; \quad \frac{30}{30} = \frac{n - 1}{0 - 1}; \quad 30 = 30\,n - 30;$$

$$n = 0 \text{ turns.}$$

If A has 30 teeth and is fixed, L 20 and the arm makes one rotation negative, then

$$e = \frac{n - a}{m - a}; \quad \frac{30}{20} = \frac{n - (-1)}{0 - (-1)}; \quad 30 = 20\,n + 20;$$

$$n = + \tfrac{1}{2} \text{ turn.}$$

This illustrates the manner in which a negative rotation of the arm is accounted for.

Fig. 123.—If A has 30 teeth and is fixed, B 40, C 20, L 40, and the arm makes one rotation positive, what are the revolutions of L?

The sign is positive and

$$e = \frac{n - a}{m - a}; \quad \frac{30}{40} \times \frac{20}{40} = \frac{n - 1}{0 - 1};$$

$$-600 = 1600\,n - 1600$$

$$n = + \tfrac{5}{8} \text{ turn.}$$

Fig. 124.—If A has 30 teeth and is fixed, B 40, C 20, L 40, and the arm makes one rotation positive, the resulting movement of L is the same as in the last example; if A be given five turns negative independently and the arm one turn negative in the same time, what are the revolutions of L?

$$e = \frac{n - a}{m - a}; \quad \frac{30}{40} \times \frac{20}{40} = \frac{n - (-1)}{-5 - (-1)};$$

$$-3000 + 600 = 1600\,n + 1600;$$

$$1600\,n = -4000; \quad n = -\tfrac{5}{2} \text{ turns.}$$

As a check by the summational method: with wheels locked and one negative turn of the arm L turns on its axis 1 negative, as also does A, but A has to make -5 turns altogether, so then -4 remain to be made up; with fixed arm and A rotating -4 times, L will make $-4 \times \frac{30}{40} \times \frac{20}{40} = -\frac{3}{2}$; total turns of L $= -1 - \frac{3}{2} = -\frac{5}{2}$.

Fig. 125.—If A has 80 teeth and is fixed, B 20, L 20, and the arm turns once positive, what are the revolutions of L?

Applying the rule for sign of e it will be found to be negative, hence

$$-e = \frac{n-a}{m-a}; \quad -\frac{80}{20} = \frac{n-1}{0-1}; \quad 80 = 20n - 20;$$

$$n = 5 \text{ turns.}$$

Fig. 126.—This arrangement is found in Moore's lifting block; the load chain is led off A and L on opposite sides, and each end does an equal share of the lifting, for the chain sprockets which are on the outside of A and L are of equal diameter, so that A and L always rotate an equal amount in opposite directions. When this condition of equal and opposite rotation is applied to the formula n and m are equal in amount but of opposite sign, then $n = -m$ or $m = -n$. The practical problem must then be, what fraction of a turn does each make for one turn of the arm.

Either A or L may be chosen as the first wheel, and e is always positive.

If A has 40 teeth, B 30, C 32, L 42, and the arm turn once positive.

$$e = \frac{n-a}{m-a}, \text{ and } m = -n, \text{ then } e = \frac{n-a}{-n-a};$$

$$\frac{40}{30} \times \frac{32}{42} = \frac{n - 1}{- n - 1};$$

$$- 1280\,n - 1280 = 1260\,n - 1260;$$

$$2540\,n = -20; \; n = -\frac{1}{127} \text{ turn,}$$

$$\text{also } m = +\frac{1}{127} \text{ turn.}$$

Fig. 127.—A and L may be upon the same spindle but must not both be keyed to it, one must be free; B is carried round by some form of rotating arm and gears in the manner shown, and is quite free to turn on its spindle. With A and L equal and A fixed, one turn round of the arm carrying B with it will cause L to turn twice in the same direction.

By the formula: if A has 30 teeth and is fixed, B 30, L 30, and the arm turn once positive, e is negative,

$$\text{therefore } - e = \frac{n - a}{m - a}; \; - \frac{30}{30} = \frac{n - 1}{0 - 1};$$

$$30 = 30\,n - 30;$$

$$n = +2 \text{ turns.}$$

Fig. 128.—If A has 30 teeth and is fixed, B 30, L 20, and the arm rotates once positive, what are the revolutions of L?

$$- e = \frac{n - a}{m - a}; \; - \frac{30}{20} = \frac{n - 1}{0 - 1}; \; 30 = 20\,n - 20;$$

$$n = +\tfrac{5}{2} \text{ turns.}$$

Fig. 129.—Here A, L and D are concentric and D is fixed, B and C are rigidly attached together and gear with D and L respectively. The arrangement is used for speed reduction, A being on a motor spindle and L on the machine to be driven.

If A has 15 teeth, B 20, and C 12, D must have

$15 + 2 \times 20 = 55$, and L $55 - (20 - 12) = 47$, and if A rotates 1000 per minute the speed of L may be found as follows:—

There are here distinctly two epicyclic trains for although the arm may be imagined fixed and with D free the gear can be turned and the relative rotations of A and L found, there still remain two unknown quantities in the equation, viz. n and a; but by treating A, B and D as one train and finding the revs. of the arm, a is found; then D, B, C, and L as another train with the same rotations of arm n can be found.

With A, B and D, D being the last wheel and fixed,

$$- e = \frac{n - a}{m - a}; \quad - \frac{15}{55} = \frac{0 - a}{1000 - a};$$

$$- 15000 + 15\,a = - 55\,a;$$

$$a = \frac{1500}{7}.$$

With D, B, C and L, calling D the first and fixed,

$$+ e = \frac{n - a}{m - a}; \quad \frac{55}{20} \times \frac{12}{47} = \frac{n - \dfrac{1500}{7}}{0 - \dfrac{1500}{7}};$$

$$6580\,n = 420000;$$

$$n = 64 \text{ turns per minute, very nearly.}$$

Fig. 130, which is a similar combination to 129 with the exception that the wheels are bevelled, is also used for reduction of speed. A is driven at high speed and the reduced motion at L is carried to the machine.

In this case the reduction of speed will be found.

If A has 16 teeth, B 48, C 24, D 60, and L 36, with

D fixed and A making one rotation, in the train A, B and D, treating D as the first wheel,

$$-e = \frac{n - a}{m - a}; \quad -\frac{60}{16} = \frac{1 - a}{0 - a}; \quad 60\,a = 16 - 16\,a;$$

$$a = \frac{4}{19} \text{ turn.}$$

In the train D, B, C and L, the arm makes $\frac{4}{19}$ turn, then with D as first wheel and fixed,

$$e = \frac{n - a}{m - a}; \quad \frac{60}{48} \times \frac{24}{36} = \frac{n - \frac{4}{19}}{0 - \frac{4}{19}}; \quad 1728\,n = \frac{1152}{19};$$

$$n = \frac{1}{28{\cdot}5} \text{ turn,}$$

or reduction of speed is 28·5 to 1.

CHAPTER XVIII

THE ODONTOGRAPH

184. Necessary Accuracy of Tooth Profile.—The method by which the profile of a tooth is set out depends upon the object in view. For ordinary illustration, where the exactness of the curve is not of primary importance, circular arcs suggesting the general form of the tooth is all that is necessary, or better still perhaps a portion of a French curve that is reasonably near to a tooth profile. For engineering draughtsmen's purposes there is really no need to draw a tooth at all; a common practice when drawing toothed wheels being to represent them by circles only, using a full line for the pitch circle and dotted lines for the addendum and dedendum circles, or a chain dotted line for the pitch circle with full lines for the addendum and dedendum circles, or some other convention equally simple, and then to make notes upon the drawing concerning the character of the teeth if such information be required. For pattern-making purposes or for testing whether an internal pinion using involute teeth will foul the wheel, the case is quite different, in both these a tooth profile as accurate as can possibly be drawn is needed.

185. Odontographs.—Many methods known as odontographs, more or less accurate, have been published, some of which use circular arcs that approach very nearly to the true curves; others use co-ordinates from a radial line by which a number of points on the true curve can be found; whilst still others use special

instruments or templates which must be placed in proper position with relation to the pitch line. The first method is the simplest and quickest, and the best of its kind is that due to G. B. Grant of the Philadelphia Gear Works and published in his treatise on gear wheels from which the details below have been extracted.

186. The Three-point Odontograph.—The three-point odontograph for a system of standard interchangeable cycloidal teeth in which the 12-toothed pinion with radial flanks is the standard, so that all rolling circles are half the diameter of this pinion, is as follows:—

THE THREE-POINT ODONTOGRAPH

STANDARD CYCLOIDAL TEETH, INTERCHANGEABLE SERIES, 10-TOOTHED PINION TO A RACK

Number of Teeth in the Wheel.		*For One Diametral Pitch.* For any other Pitch divide by that Pitch.				*For One-inch Circular Pitch.* For any other Pitch multiply by that Pitch.			
		Faces.		Flanks.		Faces.		Flanks.	
Exact.	Intervals.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.
10	10	1·99	·02	−8·00	4·00	·62	·01	−2·55	1·27
11	11	2·00	·04	−11·05	6·50	·63	·01	−3·34	2·07
12	12	2·01	·06	∞	∞	·64	·02	∞	∞
13½	13-14	2·04	·07	15·10	9·43	·65	·02	4·80	3·00
15½	15-16	2·10	·09	7·86	3·46	·67	·03	2·50	1·10
17½	17-18	2·14	·11	6·13	2·20	·68	·04	1·95	·70
20	19-21	2·20	·13	5·12	1·57	·70	·04	1·63	·50
23	22-24	2·26	·15	4·50	1·13	·72	·05	1·43	·36
27	25-29	2·33	·16	4·10	·96	·74	·05	1·30	·29
33	30-36	2·40	·19	3·80	·72	·76	·06	1·20	·23
42	37-48	2·48	·22	3·52	·63	·79	·07	1·12	·20
58	49-72	2·60	·25	3·33	·54	·83	·08	1·06	·17
97	73-144	2·83	·28	3·14	·44	·90	·09	1·00	·14
290	145-300	2·92	·31	3·00	·38	·93	·10	·95	·12
∞	Rack	2·96	·34	2·96	·34	·94	·11	·94	·11

Fig. 131 shows the application to a wheel of 18 teeth and 1″ circular pitch. The pitch, addendum, and dedendum circles are first drawn, then the lines (circles) of flank and face centres at distances according to the table, outside the pitch circle for the flank centres and inside for the face centres, and the pitch line divided into the proper spaces and thicknesses. Now with the radius from the column for flanks all the flank centres are marked off by going to each division of the pitch line in turn, and the flanks drawn

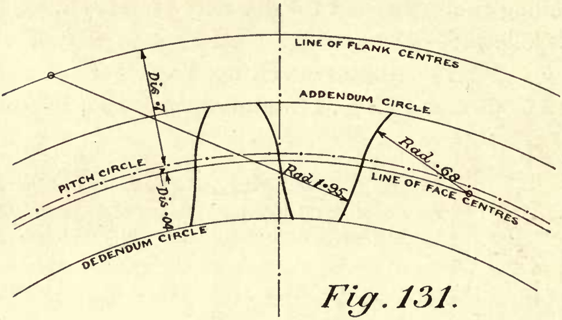


Fig. 131.

in; then readjusting the compass to the face radius, the face centres are found on their circle and the faces drawn in.

This odontograph obtains its name from the fact that the circular arcs drawn pass through three true points on the face and three on the flank, the common point on the pitch line however belonging to both.

187. The Involute Odontograph.—The involute odontograph has the centres for the approximate curves upon the base circle, so that no additional circles have to be drawn in as in the case of the cycloidal teeth. The Standard Pinion for the Interchangeable Set has obliquity of 15° and 12 teeth so

that the arc of action for two such pinions gearing together just exceeds one circular pitch.

INVOLUTE ODONTOGRAPH

Standard Interchangeable Tooth, Centres on Base Line

Teeth.	Divide by Diametral Pitch.		Multiply by Circular Pitch.	
	Face Radius.	Flank Radius.	Face Radius.	Flank Radius.
10	2·28	·69	·73	·22
11	2·40	·83	·76	·27
12	2·51	·96	·80	·31
13	2·62	1·09	·83	·34
14	2·72	1·22	·87	·39
15	2·82	1·34	·90	·43
16	2·92	1·46	·93	·47
17	3·02	1·58	·96	·50
18	3·12	1·69	·99	·54
19	3·22	1·79	1·03	·57
20	3·32	1·89	1·06	·60
21	3·41	1·98	1·09	·63
22	3·49	2·06	1·11	·66
23	3·57	2·15	1·13	·69
24	3·64	2·24	1·16	·71
25	3·71	2·33	1·18	·74
26	3·78	2·42	1·20	·77
27	3·85	2·50	1·23	·80
28	3·92	2·59	1·25	·82
29	3·99	2·67	1·27	·85
30	4·06	2·76	1·29	·88
31	4·13	2·85	1·31	·91
32	4·20	2·93	1·34	·93
33	4·27	3·01	1·36	·96
34	4·33	3·09	1·38	·99
35	4·39	3·16	1·39	1·01
36	4·45	3·23	1·41	1·03

37-40	4·20	1·34
41-45	4·63	1·48
46-51	5·06	1·61
52-60	5·74	1·83
61-70	6·52	2·07
71-90	7·72	2·46
91-120	9·78	3·11
121-180	13·38	4·26
181-360	21·62	6·88

Draw the rack by the special method.

From the table it will be seen that up to 36 teeth one radius is used for the face and another for the working flank, but above this number one radius only is necessary. Below the base line the flank is of course radial.

Fig. 132 shows the application for an equal pair of 14 teeth and 1″ circular pitch. The addendum, dedendum, pitch, and base circles being drawn in and the pitch circle divided into spaces and thicknesses, the radius for the face is taken in the compass and,

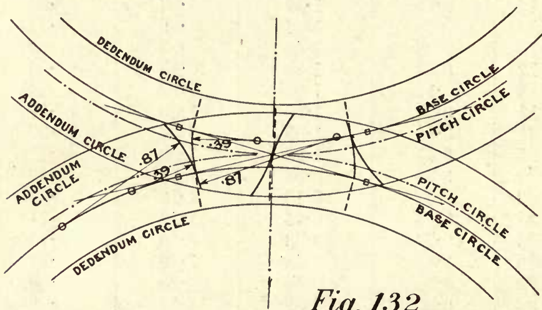


Fig. 132.

going to each division of the pitch line in turn, the faces are drawn in, the centre for the arc being upon the base circle; readjusting the compass to the flank radius the curved parts of the flanks are inserted and afterwards the radial portion.

The table of figures contains reference to 10 and 11-toothed pinions; these, although theoretically possessing too few teeth the arc of contact being less than the circular pitch, yet practically gear with fairly smooth motion.

188. The Involute Rack Odontograph.—A special construction is needed for the rack because in a system

of involute teeth of ordinary proportions in which 12 is a minimum the rack interferes with the smaller wheels ; to avoid this interference the face beyond the interference point is rounded off with a cycloid.

Fig. 133 illustrates the method pursued which is as follows : the sides of the rack teeth are drawn as straight lines at 15° to the line of centres, but the outer half of the face is made a circular arc of radius 2·1″ divided by the diametral pitch, or ·67″ multiplied by the circular pitch, the centre for the arc being on the pitch line.

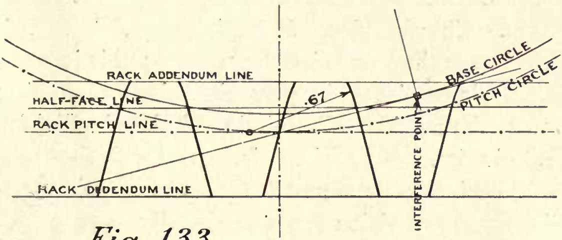


Fig. 133.

189. Method of Rolling Gear Teeth Profiles.— In the absence of a good odontographic instrument or a table of figures suited to any particular system of tooth formation, the profile must be set out by the best geometrical method known to the operator. A very old and a very useful scheme is by direct rolling, using a piece of tracing paper to carry the rolling circle for cycloidal teeth or the tangent line to the base circle for involute teeth, and two finely pointed needles for securing the several positions of the rolling piece and the track of the tracing point ; the resulting accuracy depending upon the skill and the care exercised in manipulating the pieces.

The process for cycloidal teeth is as follows: having the sizes of the rolling circles, draw about a half circle of each on separate pieces of tracing paper and prick off with the aid of a pair of spring dividers a number of equal parts about $\frac{1}{4}''$ long, on the circumference of each, and number them in order 0, 1, 2, 3, etc. Now divide the pitch circle of the wheel into the same equal parts, commencing from say the pitch point work in both directions and number them oppositely as in Fig. 134. Draw radial lines through the divisions of all the circles, and extend the radials of the outside rolling circle about $\frac{1}{2}$ to $\frac{3}{4}$ a radius length. Cut the pieces of tracing paper down to a little larger than the diagram upon it; this is to reduce the loose pieces to a convenient size for handling.

Take the outside rolling circle and place its division 1 on division 1 of the pitch circle, and with the radial lines in coincidence secure it with a needle point, then prick through division 0, not too heavily so as to open the hole already there but just sufficient to leave its mark on the paper below. Release and adjust similarly at 2, and again prick through 0. Repeat until the track of 0 reaches a little beyond the addendum circle. A smooth curve through the points found gives the face profile.

In Fig. 134 the outer rolling circle is shown in coincidence at 4 and the marked positions of 0 are indicated by small circles.

Treatment of the inner rolling circle in the same manner, but on the other side of the starting point 0, will trace out the flank profile. In the figure coincidence is shown at 5.

190. Accuracy of the Rolling Process.—The ra-

dial lines are not necessary to the construction but are an aid in determining the position of tangency of the

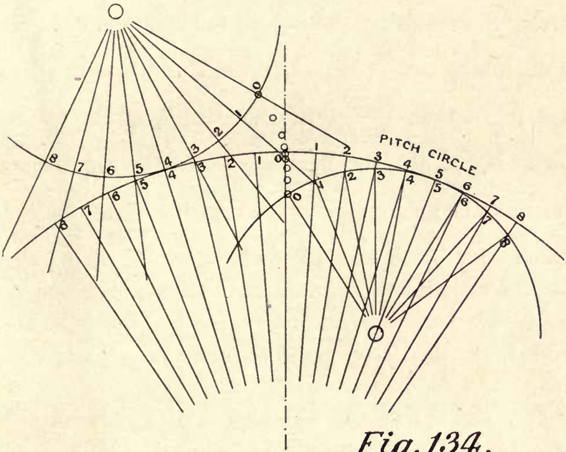


Fig. 134.

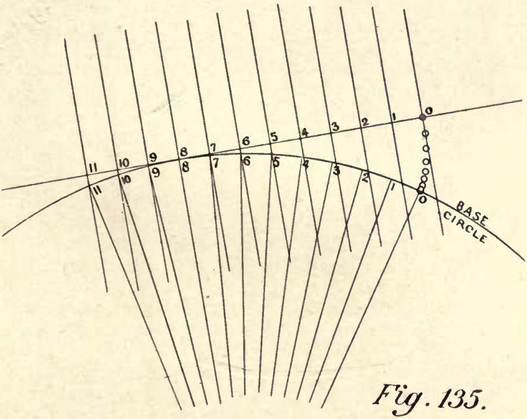


Fig. 135.

several circles, and by their proper use a more accurate result is likely to accrue. The actual length of

the divisions chosen rests with the operator. If the circles are large enough to use 5° of arc of the rolling circle, the resulting theoretical error is about $\frac{1}{50000}$ of the rolling circle radius for a small numbered pinion, and for a rack $\frac{6 \text{ to } 7}{10000}$ of the rolling circle radius. Using 10° of arc, the maximum error in the case of the rack is about $\frac{1}{200}$ of the rolling circle radius, which may be looked upon as the worst case, for if the dimensions are so small that divisions larger than 10° of arc must be chosen the method must be discarded, or the tooth profile drawn to an enlarged scale.

Fig. 135 shows the application of the scheme to involute teeth. The method of operating is the same as for cycloidal teeth and requires no further explanation, except to note that the rolling of the tangent line is upon the base circle, and the radials of the rolling circles of Fig. 134 have become lines at right angles to the tangent in Fig. 135.

THE END

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